Any *algorithm* for a problem $\mathcal{P}$, (i) halts for every input and (ii) produces correct output for every input according to the description in $\mathcal{P}$. For the following well-known problems there cannot exist any algorithms since they are proven to be undecidable:

- *halting problem*: no algorithm can determine whether a program P halts on an input w; here, both P and w are given as inputs to possible algorithm

- *program correctness*: no algorithm that can determine whether any given program will always produce the correct output for all possible inputs

- *Godel's first incompleteness theorem*: in any reasonable system capable of expressing basic arithmetic will contain some true statements unprovable within that system

- *busy beaver*: deciding whether a given TM is the longest-running among halting TMs with the same number of states and symbols

- *Kolmogorov complexity of a string*: for any binary string x, the minimal description $d(x)$ of x is the shortest string $< M, w >$, where TM M on input w halts with x on its tape (if several such strings exist, $d(x)$ is the string that occurs first in the canonical ordering among them); given x and ℓ, determining whether the Kolmogorov complexity $|d(x)|$ of x is ℓ

- *Hilbert's tenth problem*: for any given polynomial equation E with integer coefficients and a finite number of unknowns, decide whether E has an integer solution

- *mortal matrix problem*: given a finite set S of $n \times n$ matrices with integer entries, decide whether the zero matrix can be expressed as a finite product of matrices from S, for $|S| \geq 6$ and $n \geq 3$

- *a ray tracing problem*: for a 3-dimensional system of reflective or refractive objects, deciding if a ray beginning at a given position and direction eventually reaches a certain point

- deciding whether a particle of an ideal fluid in a three dimensional domain eventually reaches a certain region in space

- deciding whether two input *finite simplicial complexes are homeomorphic*, that is, continuously deformed into each other without tearing or gluing

References:
- wiki