Region Elimination Methods

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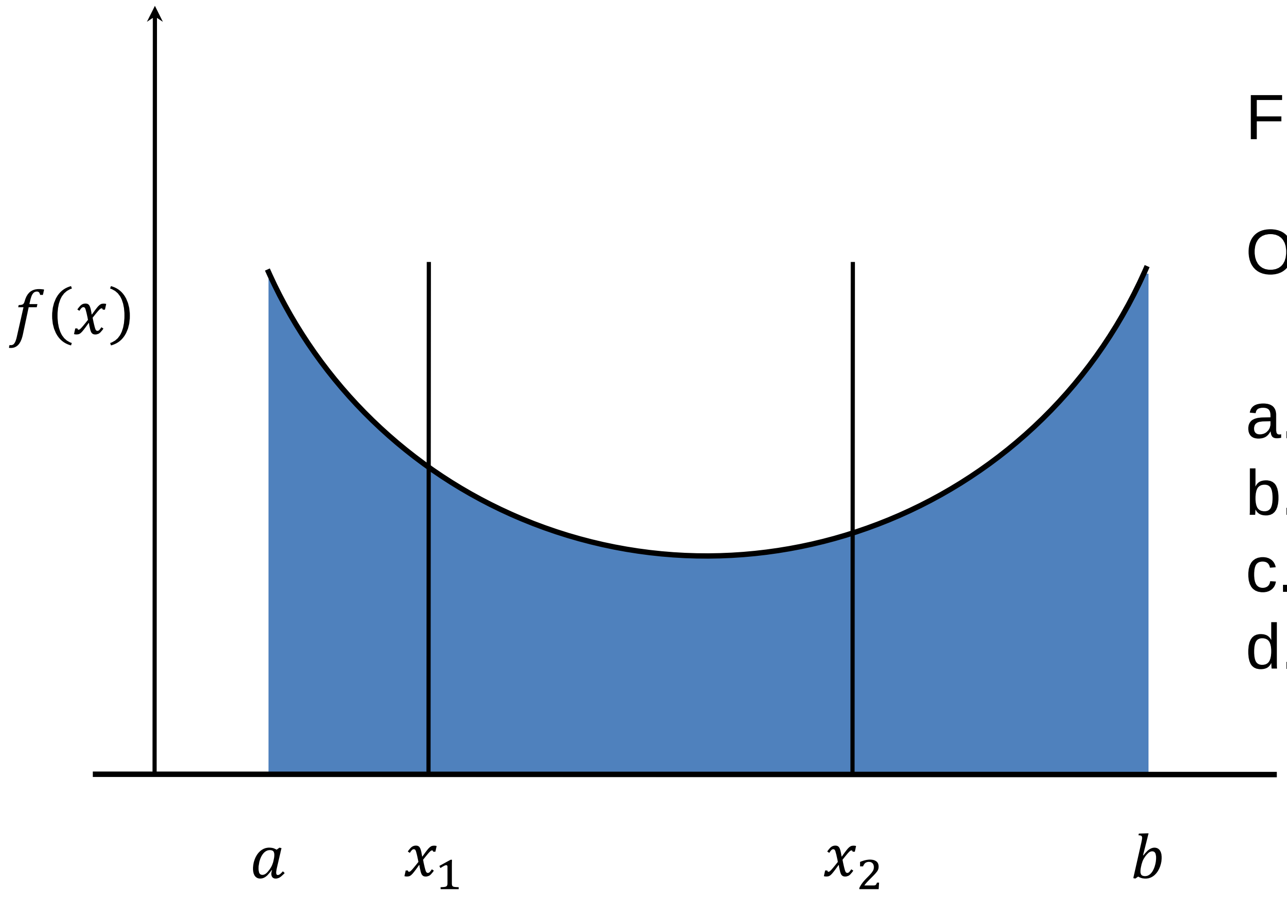
Region Elimination Method



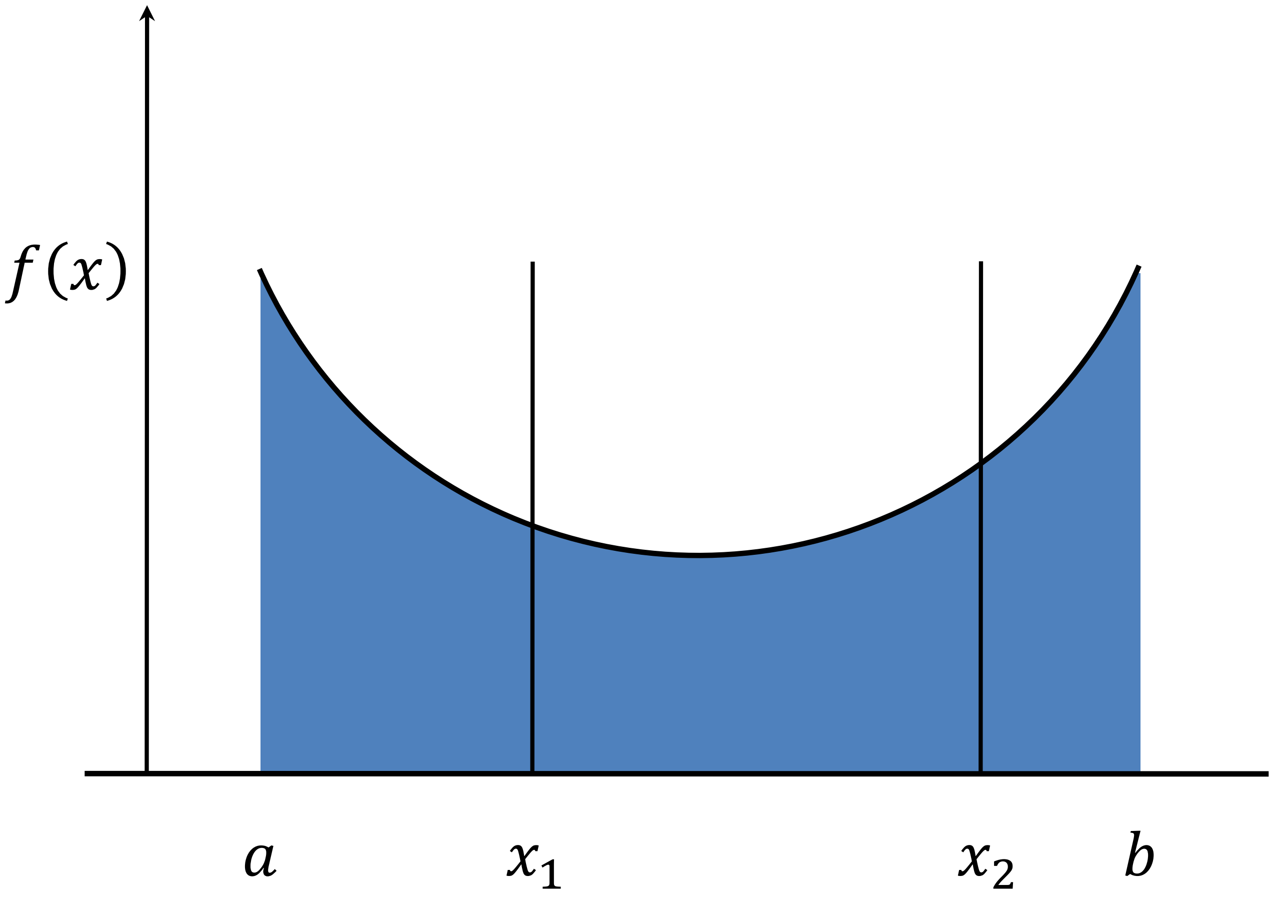
For the unimodal function $if\ f(x_1) > f(x_2)$

Optimum is not between

a. Between $[a, x_1]$
b. Between $[x_1, x_2]$
c. Between $[x_2, b\ ]$
d. Between $[a, b\ ]$

Ans: a

Region Elimination Method



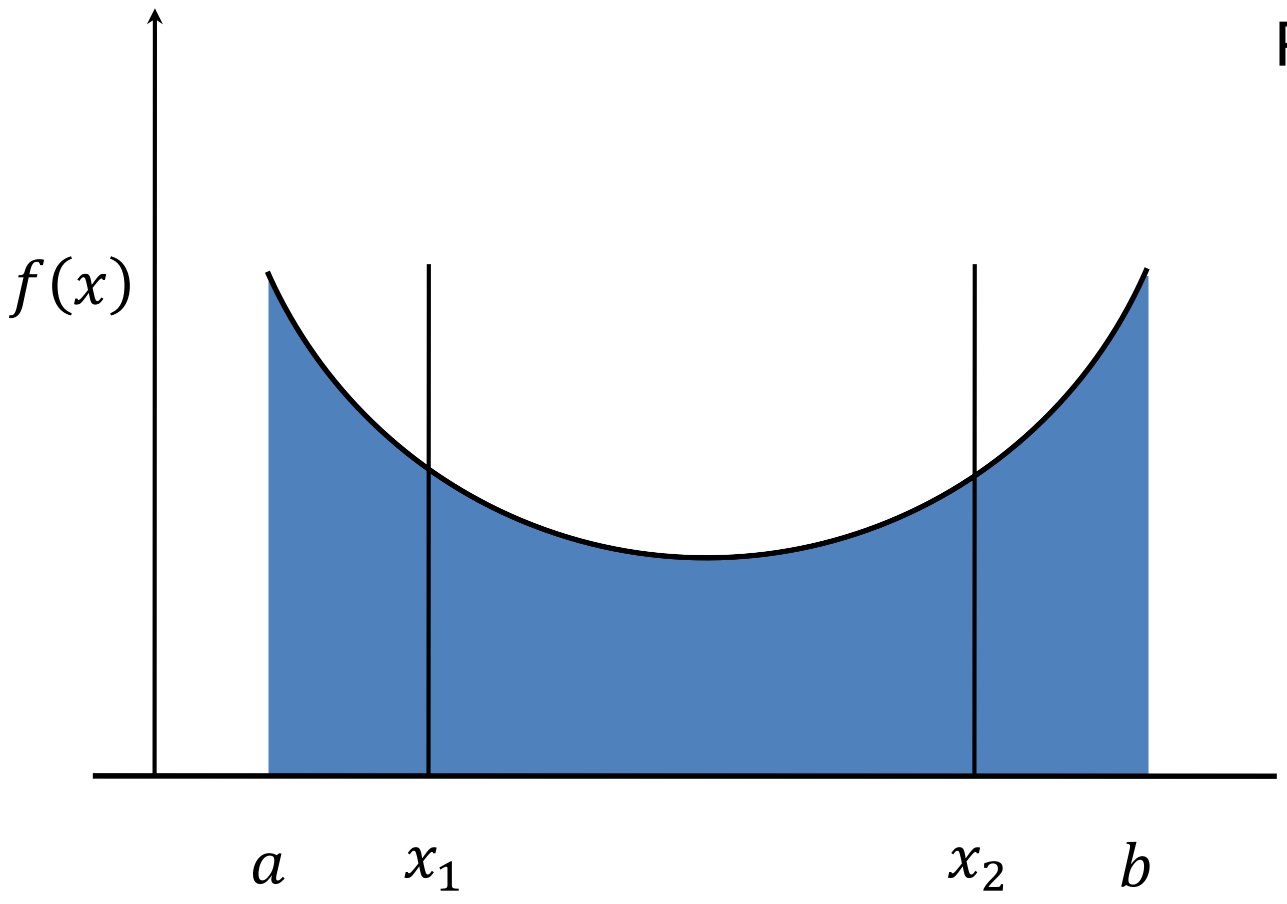
For the unimodal function $if\ f(x_2) > f(x_1)$

Optimum is not there between

a. Between $[a, x_1]$
b. Between $[x_1, x_2]$
c. Between $[x_2, b\ ]$
d. Between $[a, b\ ]$

Ans: c

Region Elimination Method



For the unimodal function $if\ f(x_1) = f(x_2)$

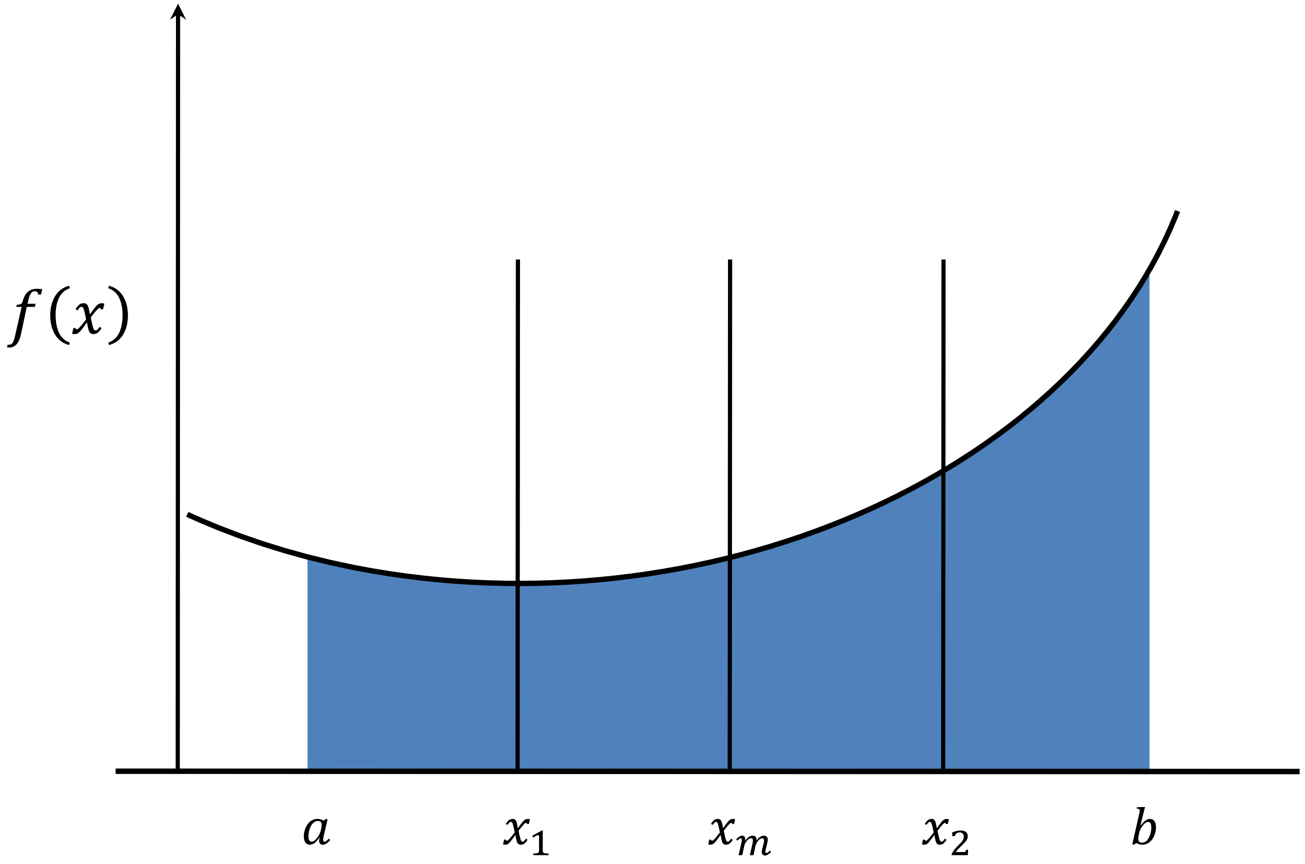
Optimum is not there between

a. Between $[a, x_1]$
b. Between $[x_1, x_2]$
c. Between $[x_2, b\ ]$
d. Between $[a, b\ ]$

Ans: a, c

Region Elimination Method

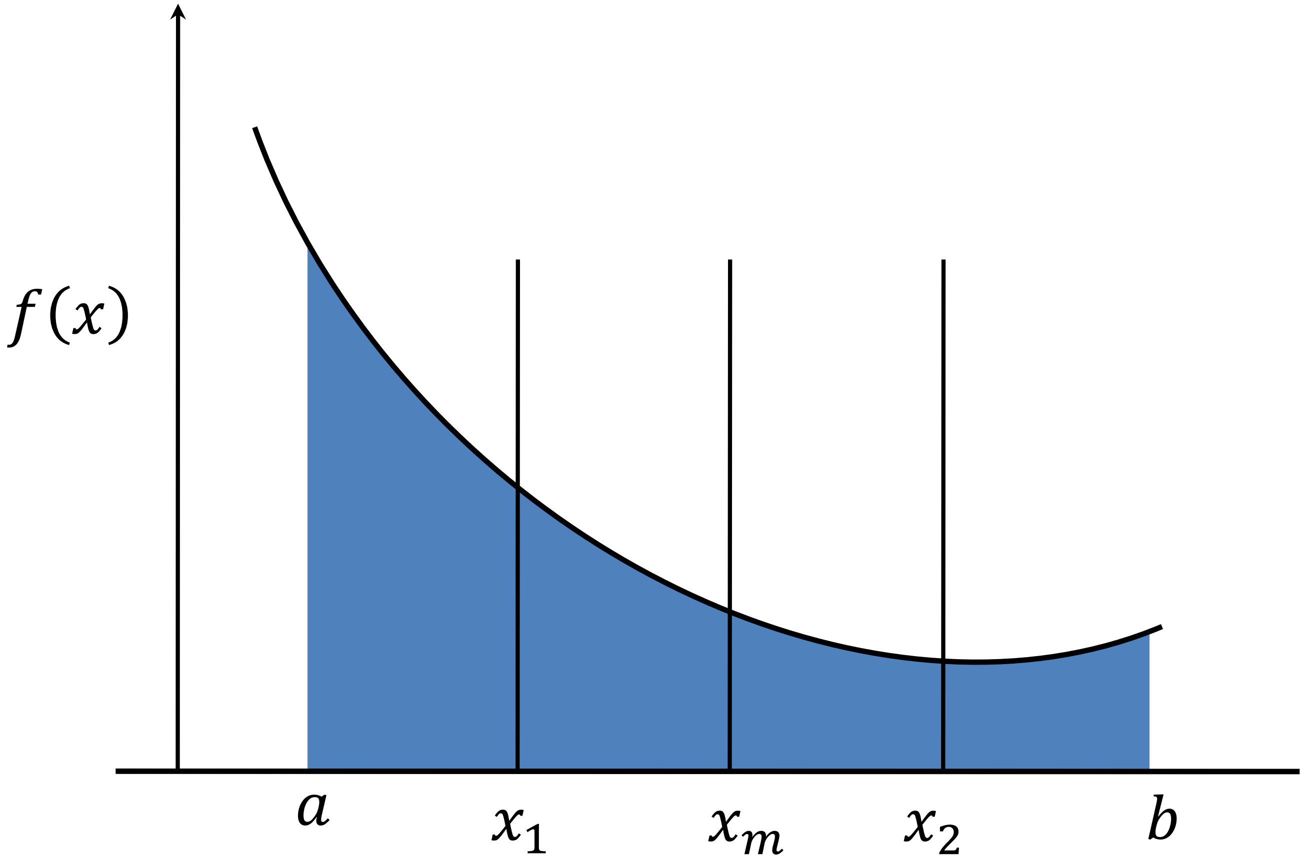
Interval halving method



$$if\ f(x_1) < f(x_m)$$

Region Elimination Method

Interval halving method

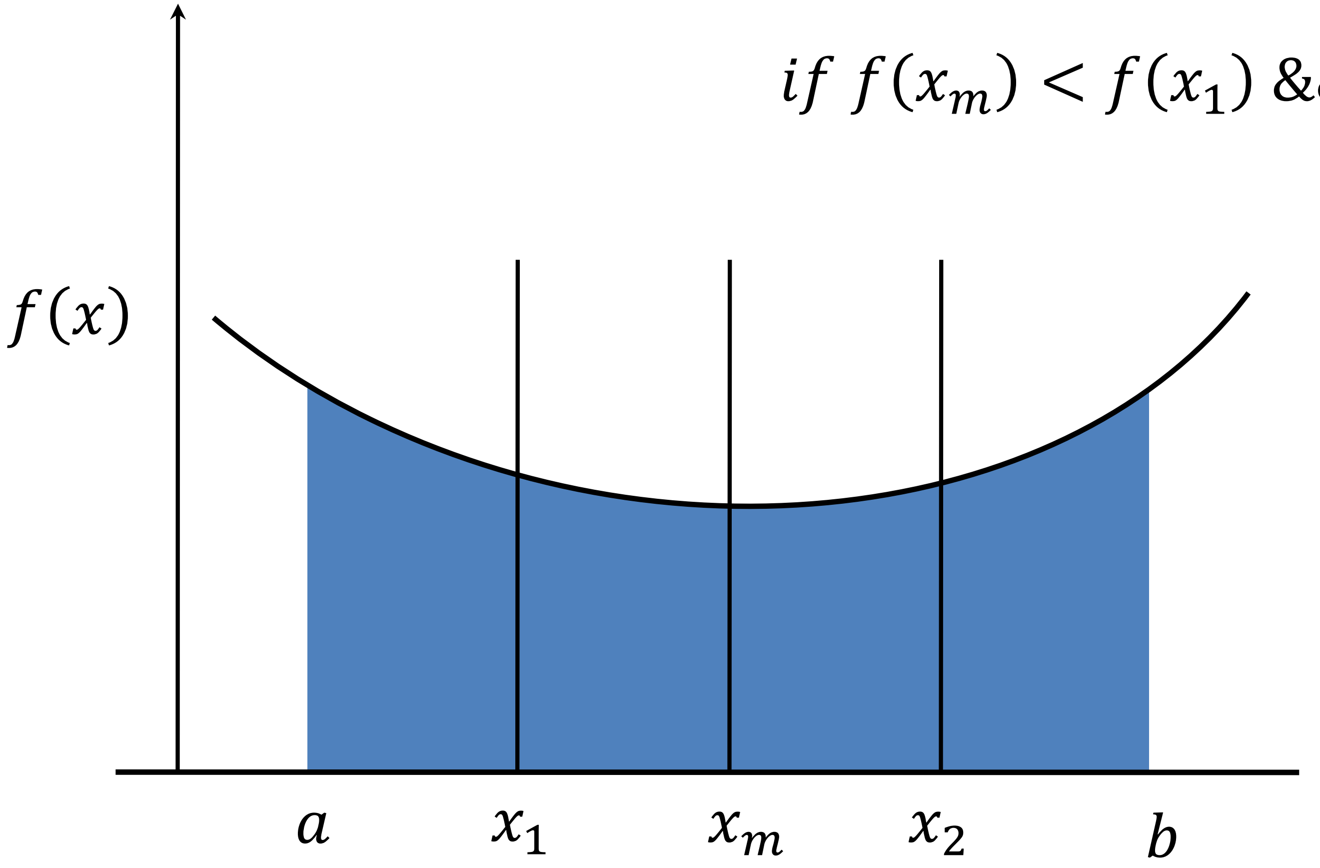


$if\ f(x_2) < f(x_m)$

Region Elimination Method

Interval halving method

$if\ f(x_m) < f(x_1)\ \&\& f(x_m) < f(x_2)$

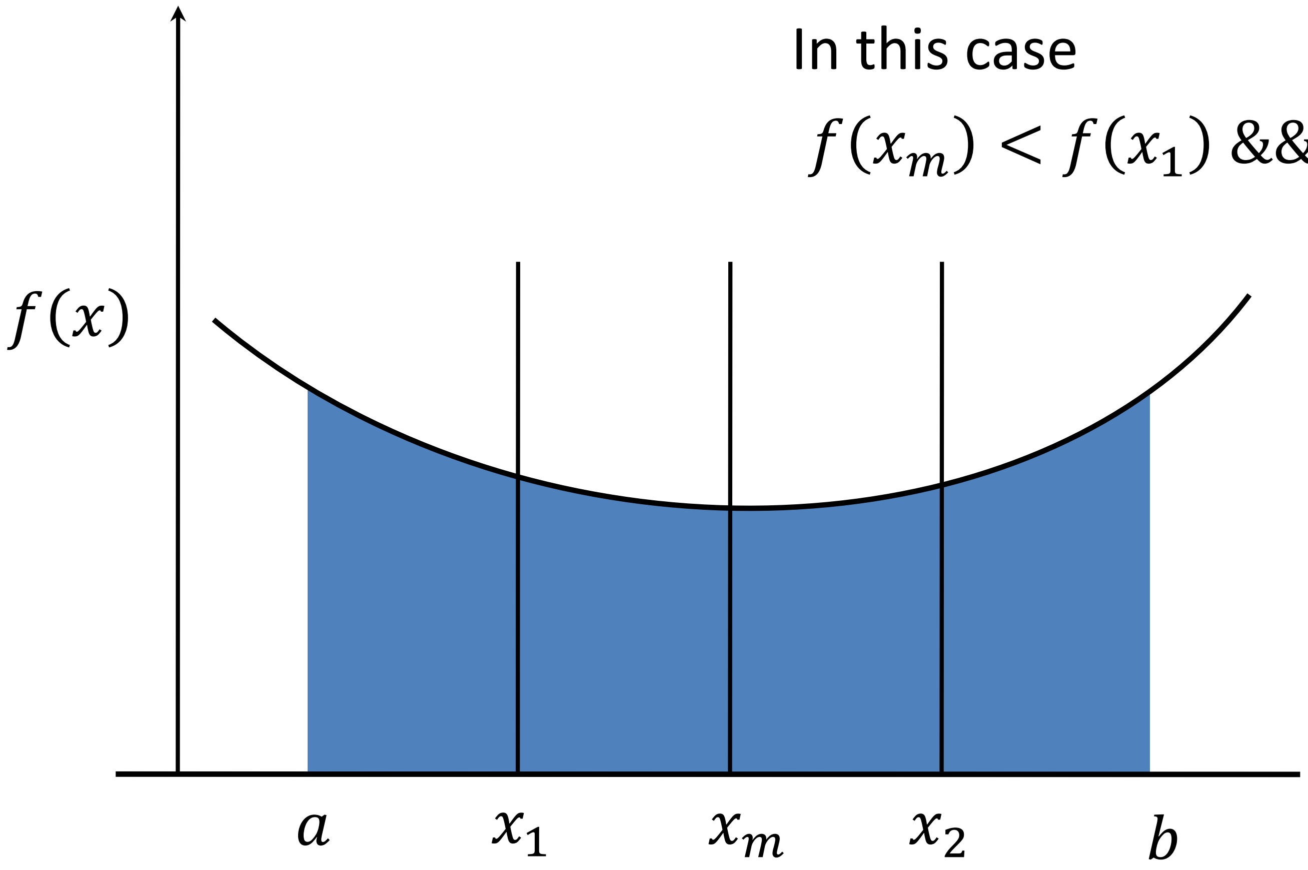


Region Elimination Method

Interval halving method

In this case

$$f(x_m) < f(x_1) \;\&\&\; f(x_m) < f(x_2)$$

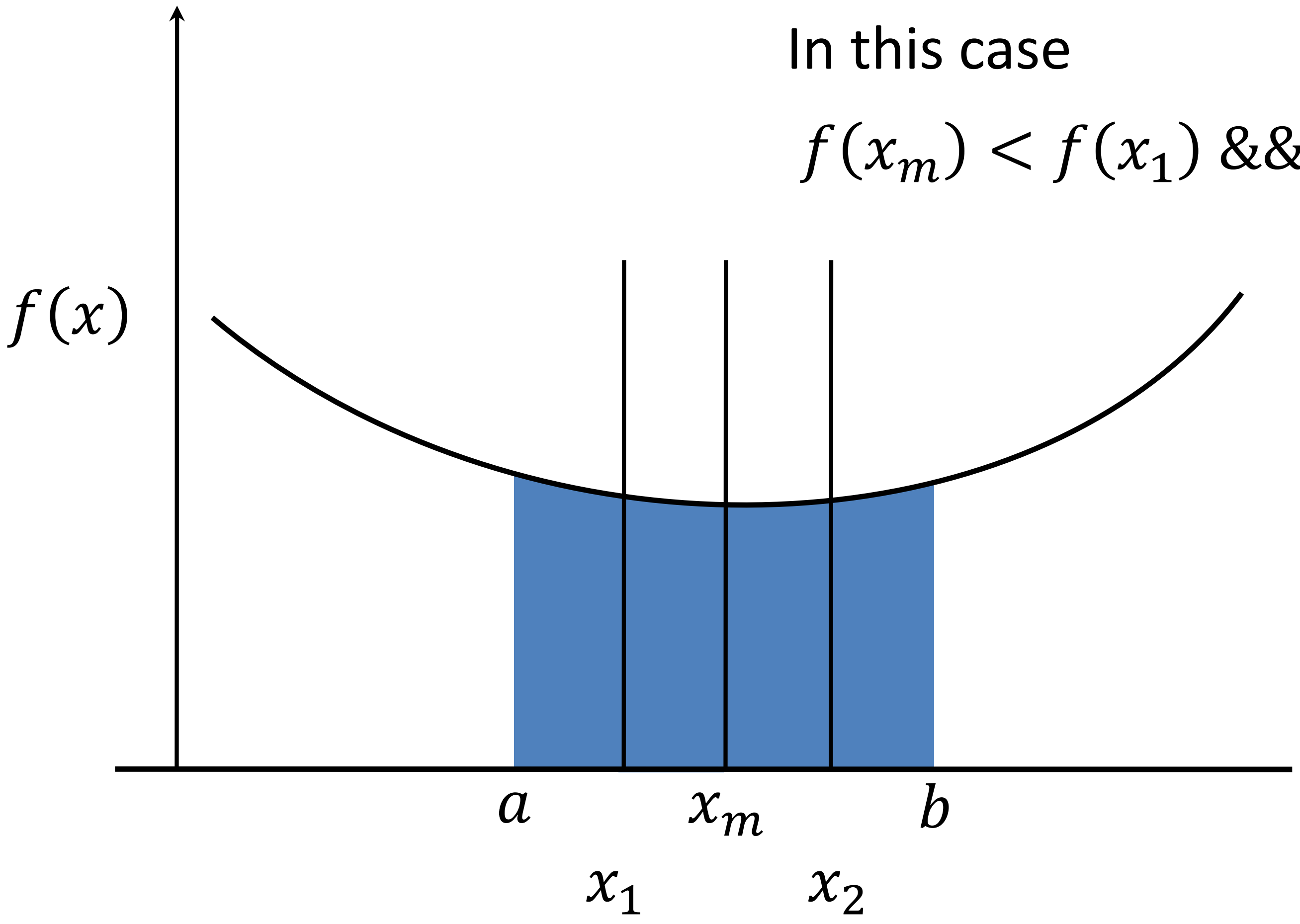


Region Elimination Method

Interval halving method

In this case

$$f(x_m) < f(x_1)\ \&\& f(x_m) < f(x_2)$$



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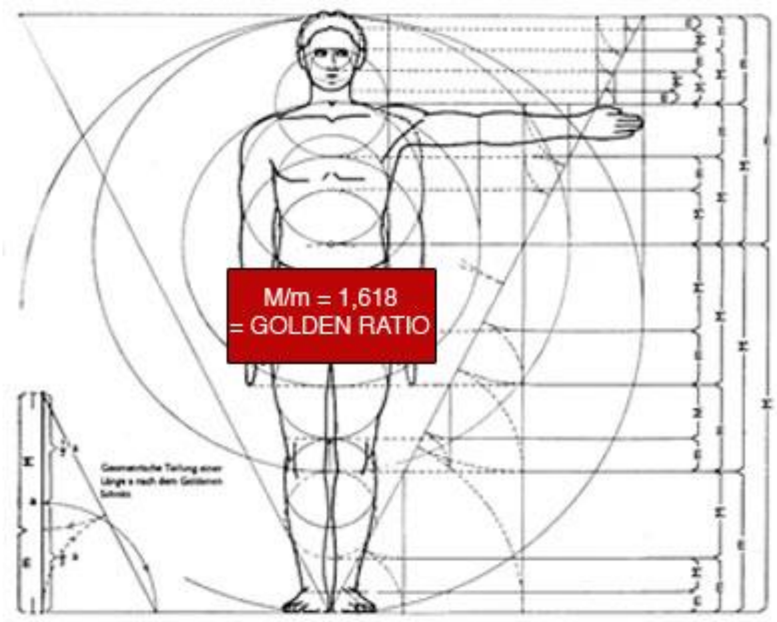
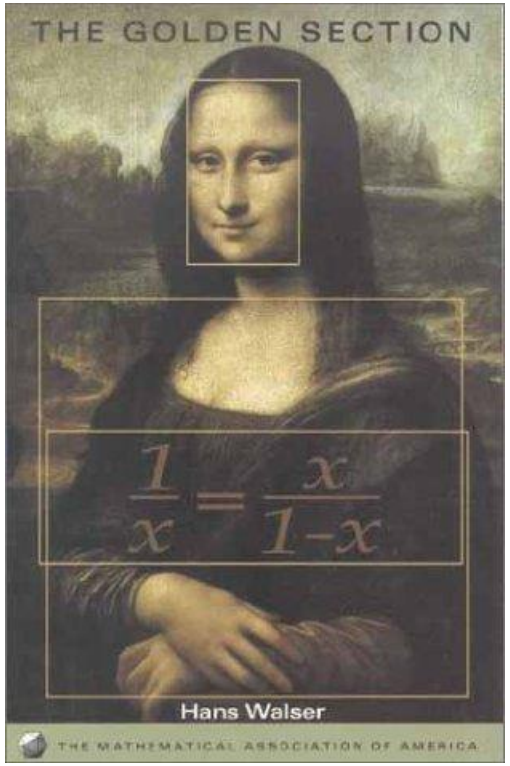
Region Elimination Method

Golden Section Search Method

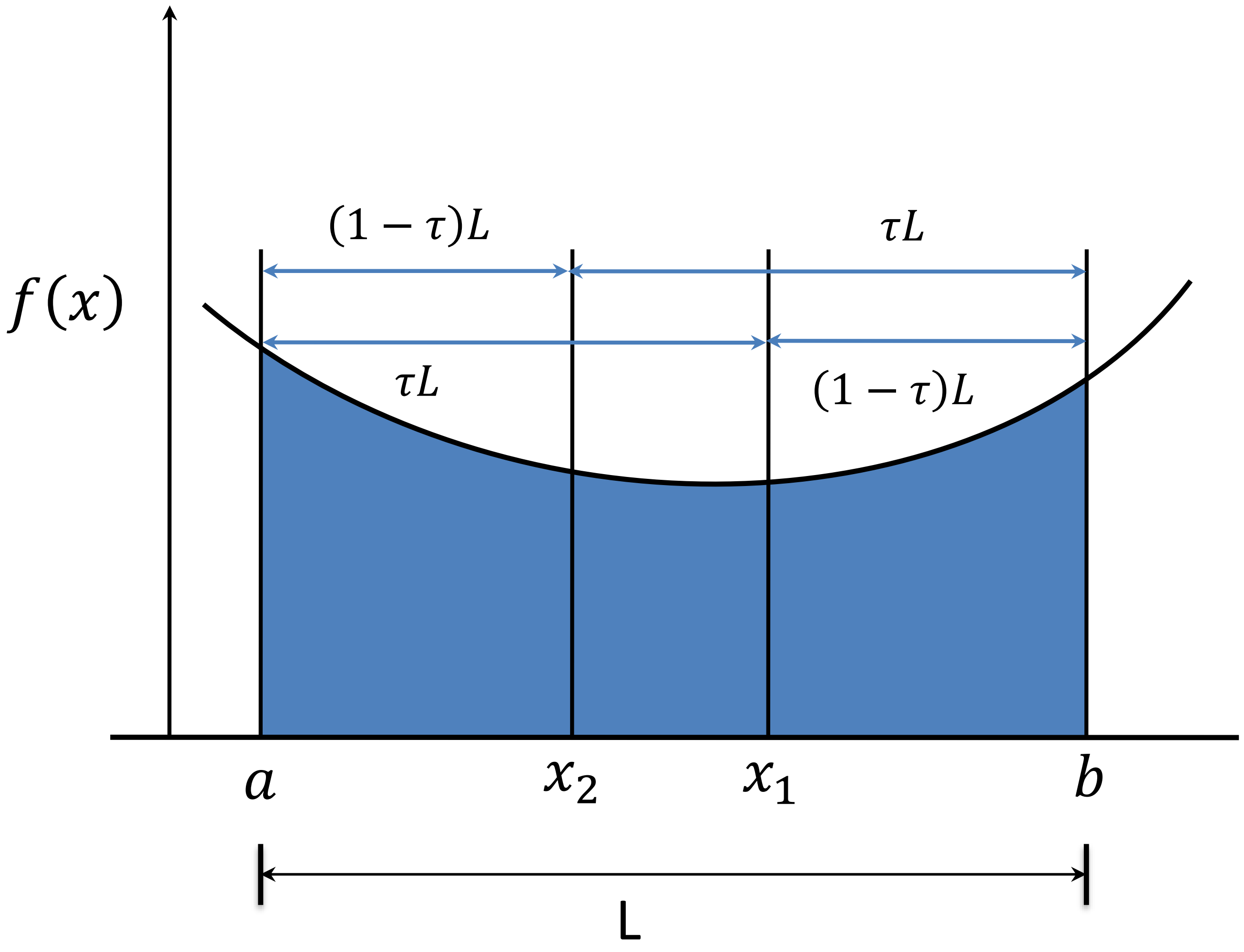




Golden ratio 0.618=1/1.618

Region Elimination Method

Golden Section Search Method



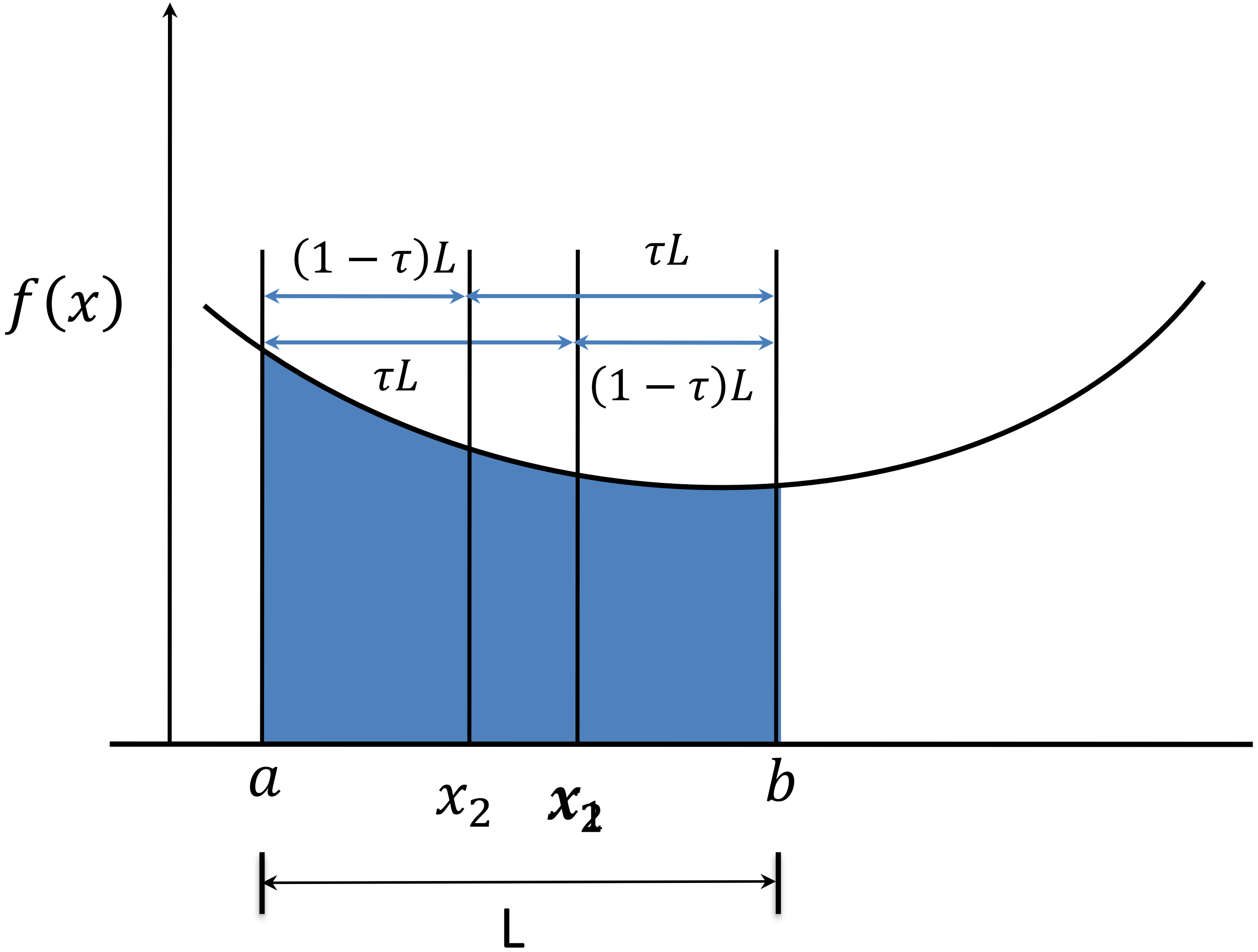
Apply region elimination rules

Suppose

$$f(x_1) > f(x_2)$$

Region Elimination Method

Golden Section Search Method



Apply region elimination rules

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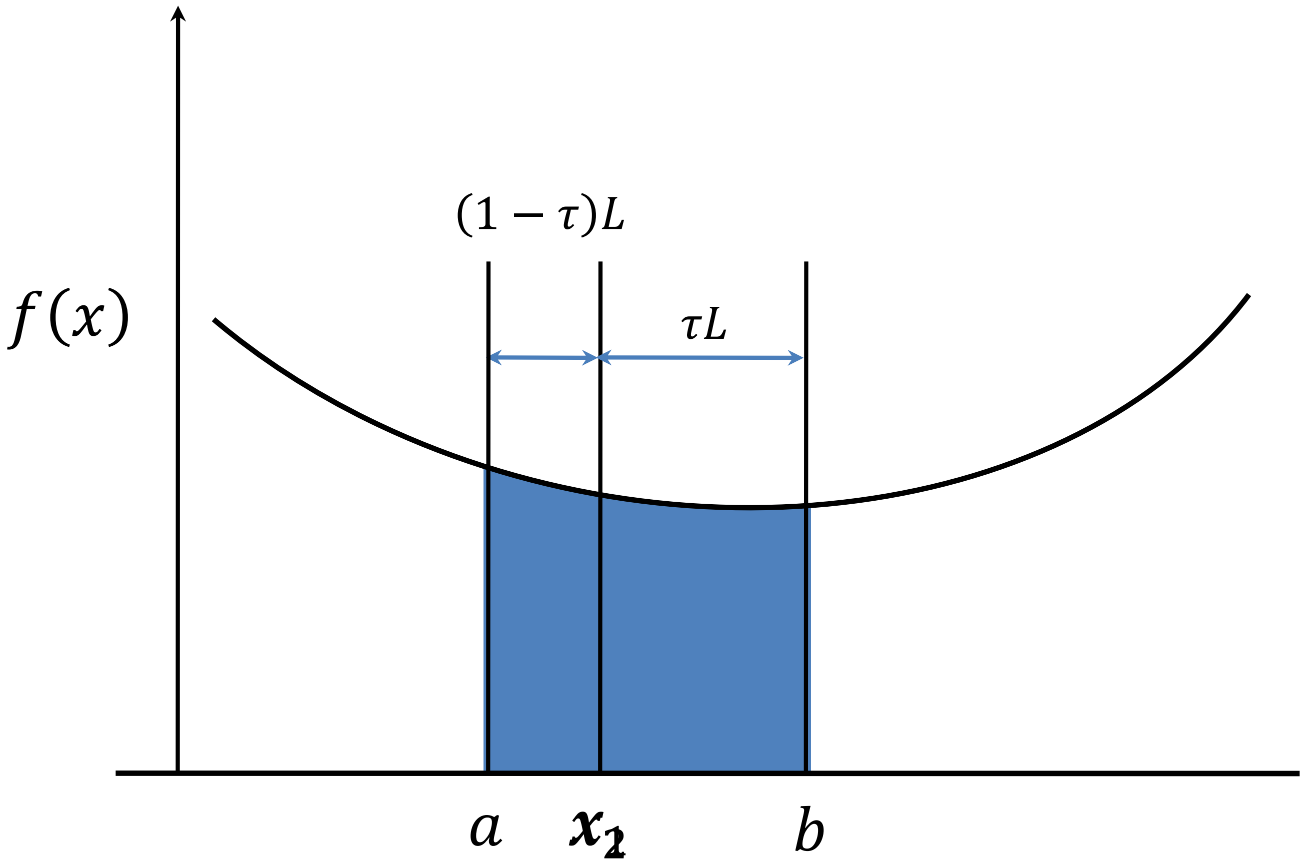
$$f(x_1) < f(x_2)$$

Region Elimination Method

Golden Section Search Method



Continue iteration

Region Elimination Method

Golden Section Search Method

$$c = a + \tau(b - a) \qquad (1)$$

$$d = b - \tau(b - a) \qquad (2)$$

If $f(d) < f(c)$

$$d = a + \tau(c - a) \qquad (3)$$

Putting (1) in (3), we have

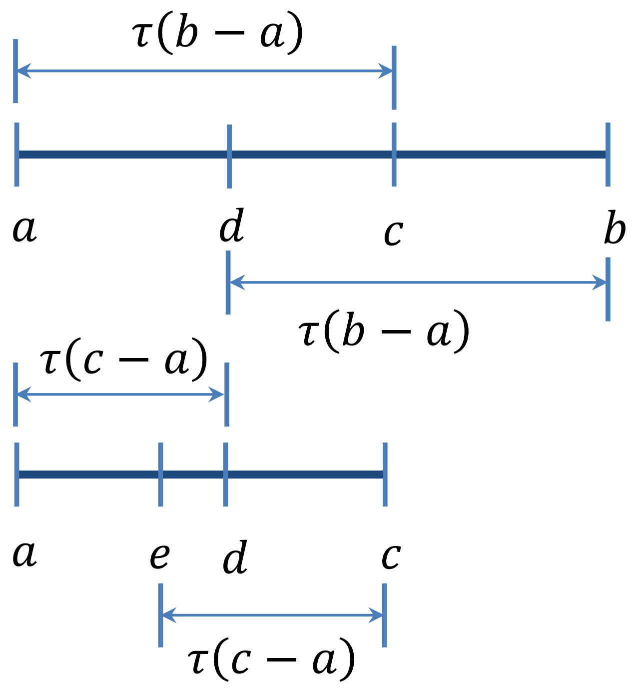
$$d = a + \tau(a + \tau(b - a) - a)$$

$$d = a + \tau^2(b - a) \qquad (4)$$

Equating (4) and (2), we have

$$b - \tau(b - a) = a + \tau^2(b - a)$$

$\tau^2 + \tau - 1 = 0$ Solving τ=0.618, -1.618



0.618 is the golden

Region Elimination Method

QUIZ

1. If $f(x)$ is an unimodal convex function in the interval $[a, b]$, then $f'(a) \times f'(b)$ is

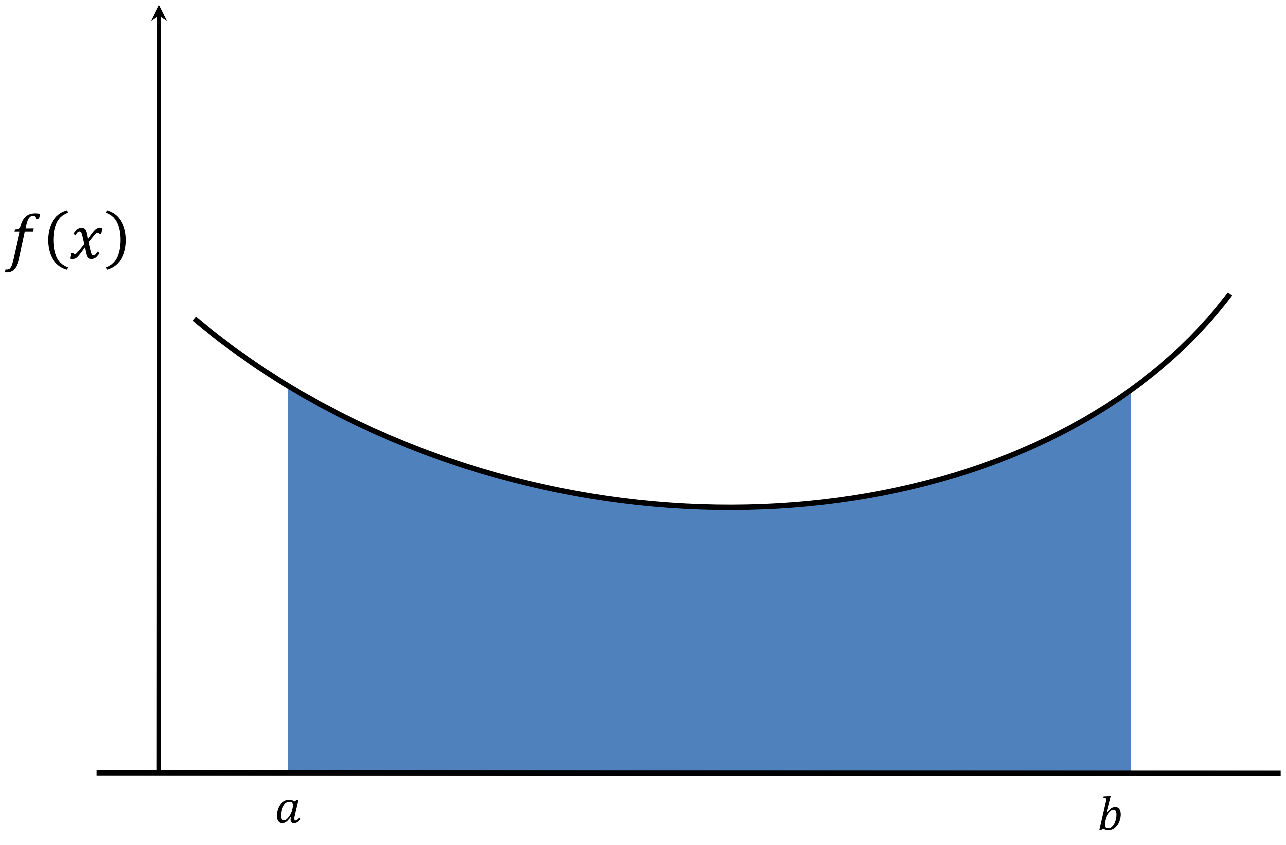
a) Positive
b) Negative
c) It may be negative or may be positive
d) None of the above

2. For the same function, take any point c between $[a, b]$. If $f'(c)$ is less than 0, then minimum does not lie in

a) $[a, c]$
b) $[c, b]$
c) $[a, b]$
d) None of the above

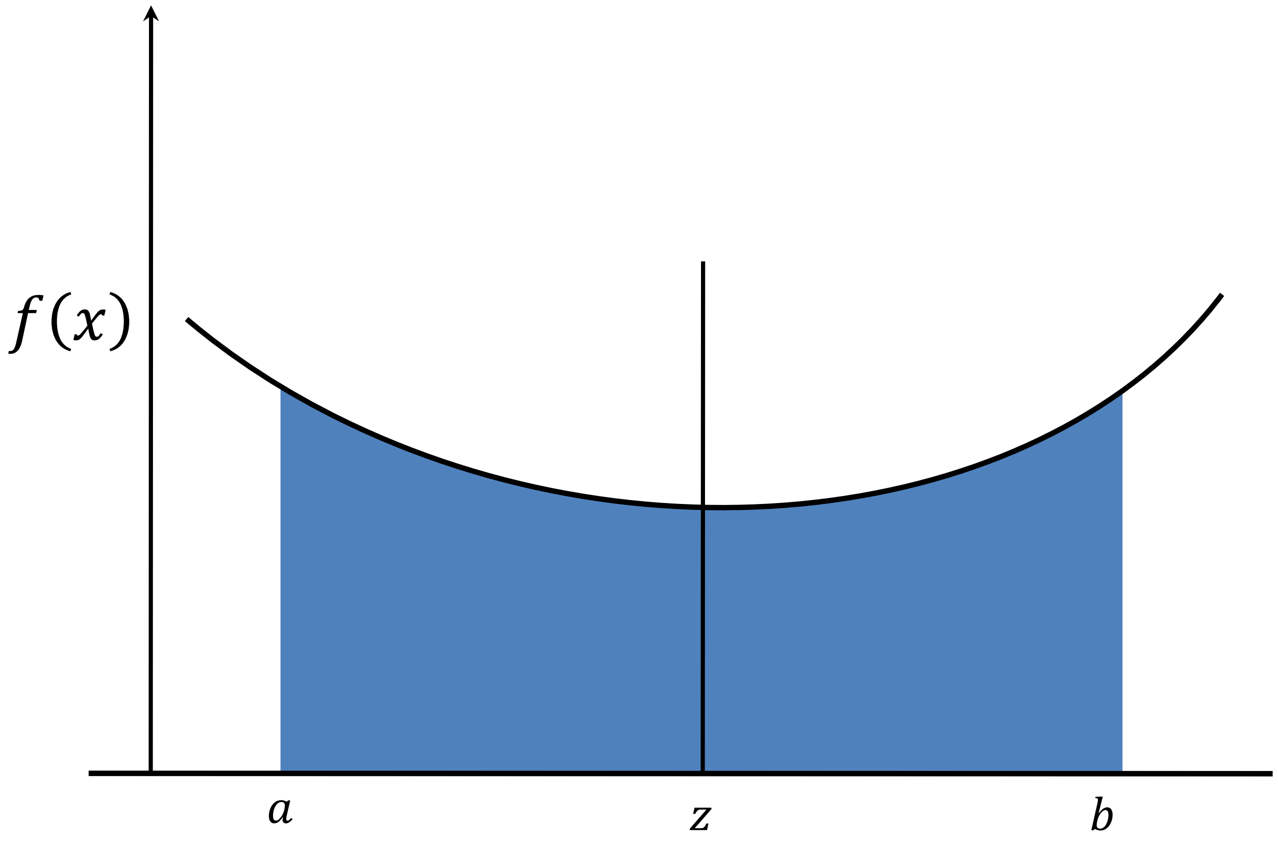
2. For the same function, take any point c between $[a, b]$. If $f'(c)$ is greater than 0, then minimum does not lie in

a) $[a, c]$
b) $[c, b]$
c) $[a, b]$
d) None of the above



Region Elimination Method

Bisection method



Take a point $z = \frac{a+b}{2}$

$if\, f'(z) < 0$ then area between $[a, z]$ will be eliminated

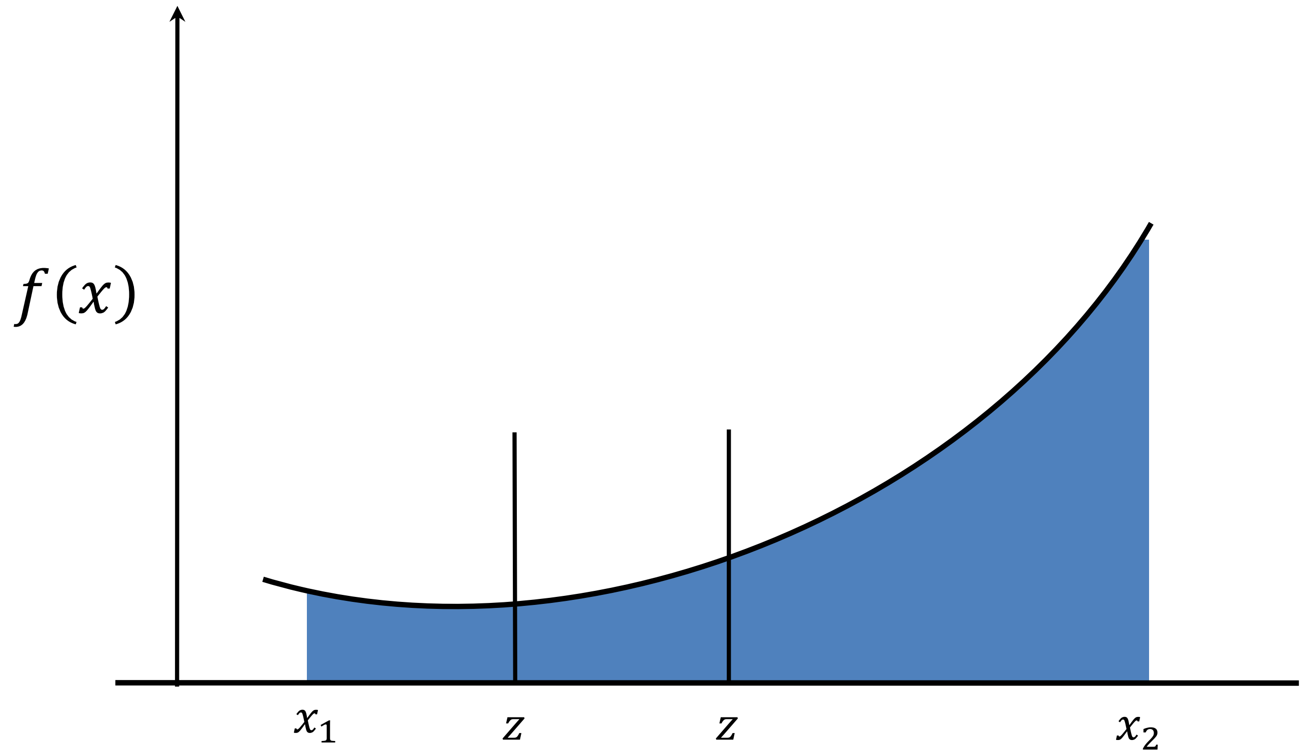
$if\, f'(z) > 0$ then area between $[z, b]$ will be eliminated

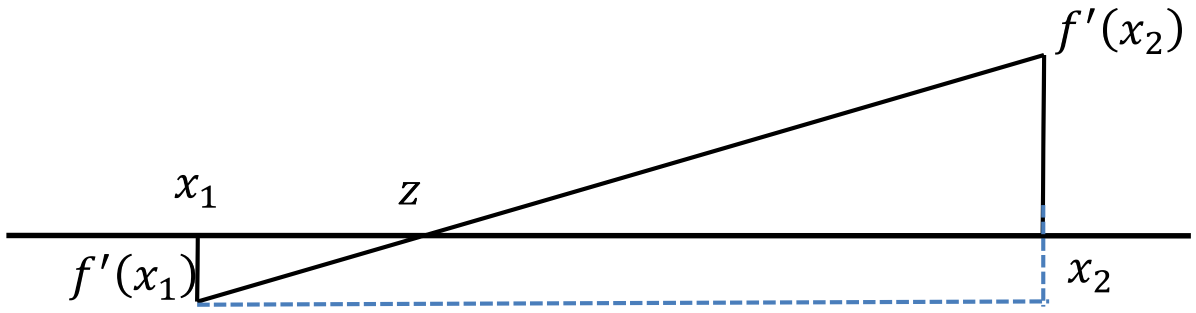
Disadvantage

- Magnitude of the derivatives is not considered

Region Elimination Method

Bisection method





Apply region elimination technique

In this case $f'(z) > 0$

then area between $[z, x_2]$ will be eliminated

Considering similar triangle

$$\frac{f'(x_2)}{x_2 - z} = \frac{f'(x_2) - f'(x_1)}{x_2 - x_1}$$

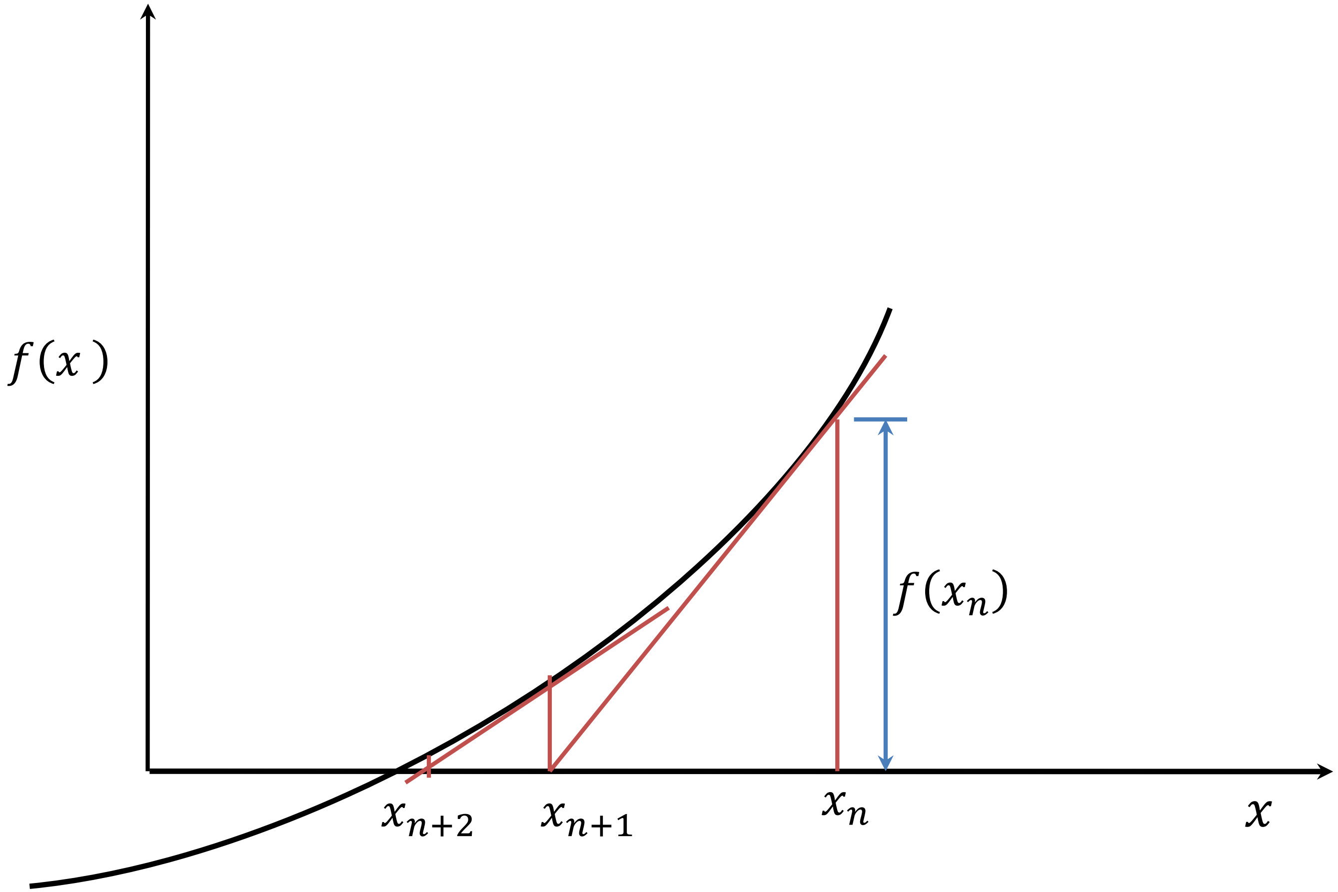
$$z = x_2 - \frac{f'(x_2)}{\frac{f'(x_2) - f'(x_1)}{x_2 - x_1}}$$

Region Elimination Method

Thanks

Region Elimination Method

Newton-Raphson method



$$f'(x_n) = \frac{f(x_n) - f(x_{n+1})}{x_n - x_{n+1}}$$

Rearranging and putting $f(x_{n+1}) = 0$

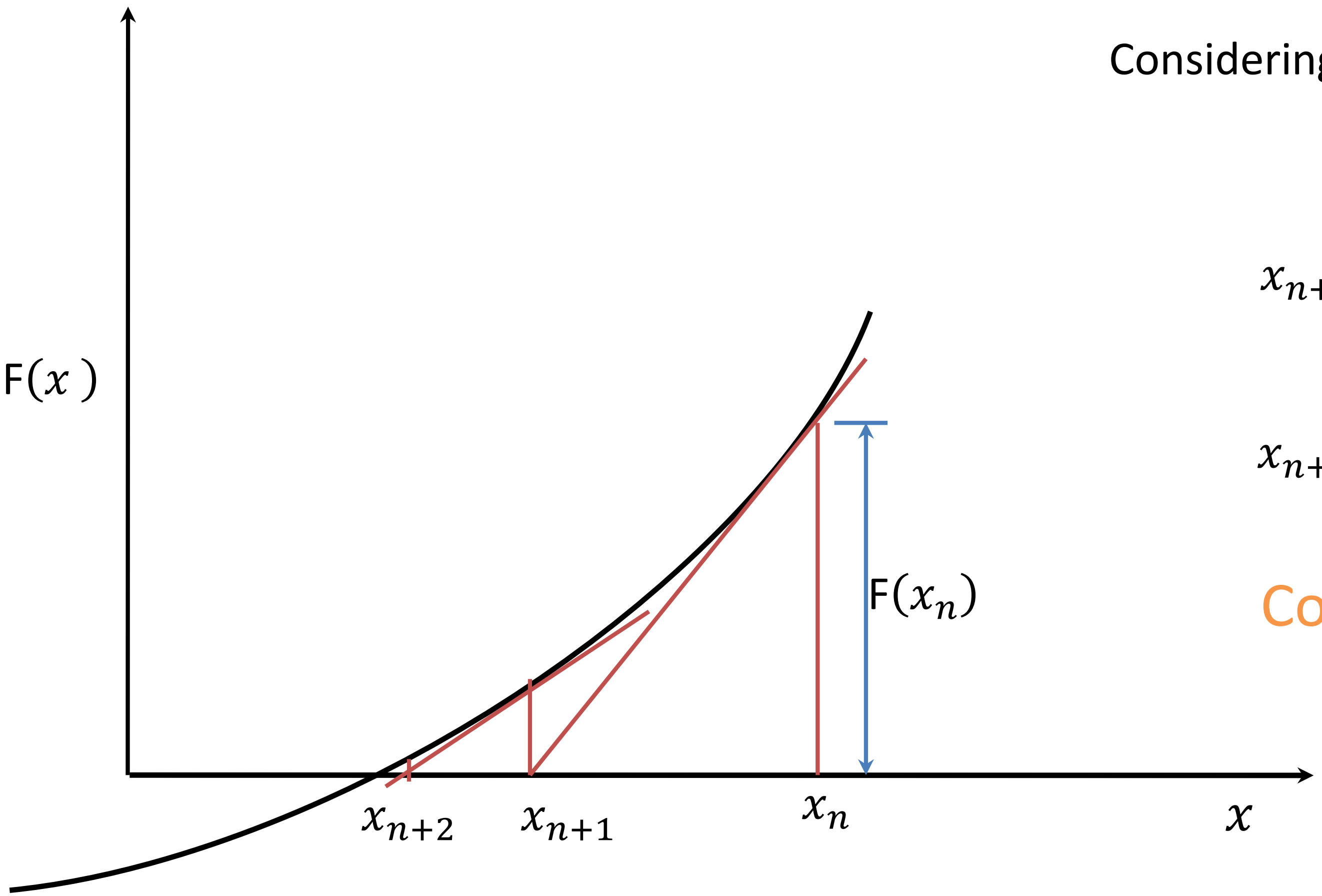
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+2} = x_{n+1} - \frac{f(x_{n+1})}{f'(x_{n+1})}$$

Continue iteration

Region Elimination Method

Newton-Raphson method



Incase optimization problem, $f'(x) = 0$

Considering $F(x) = f'(x)$

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

Continue iteration