Line Search Methods

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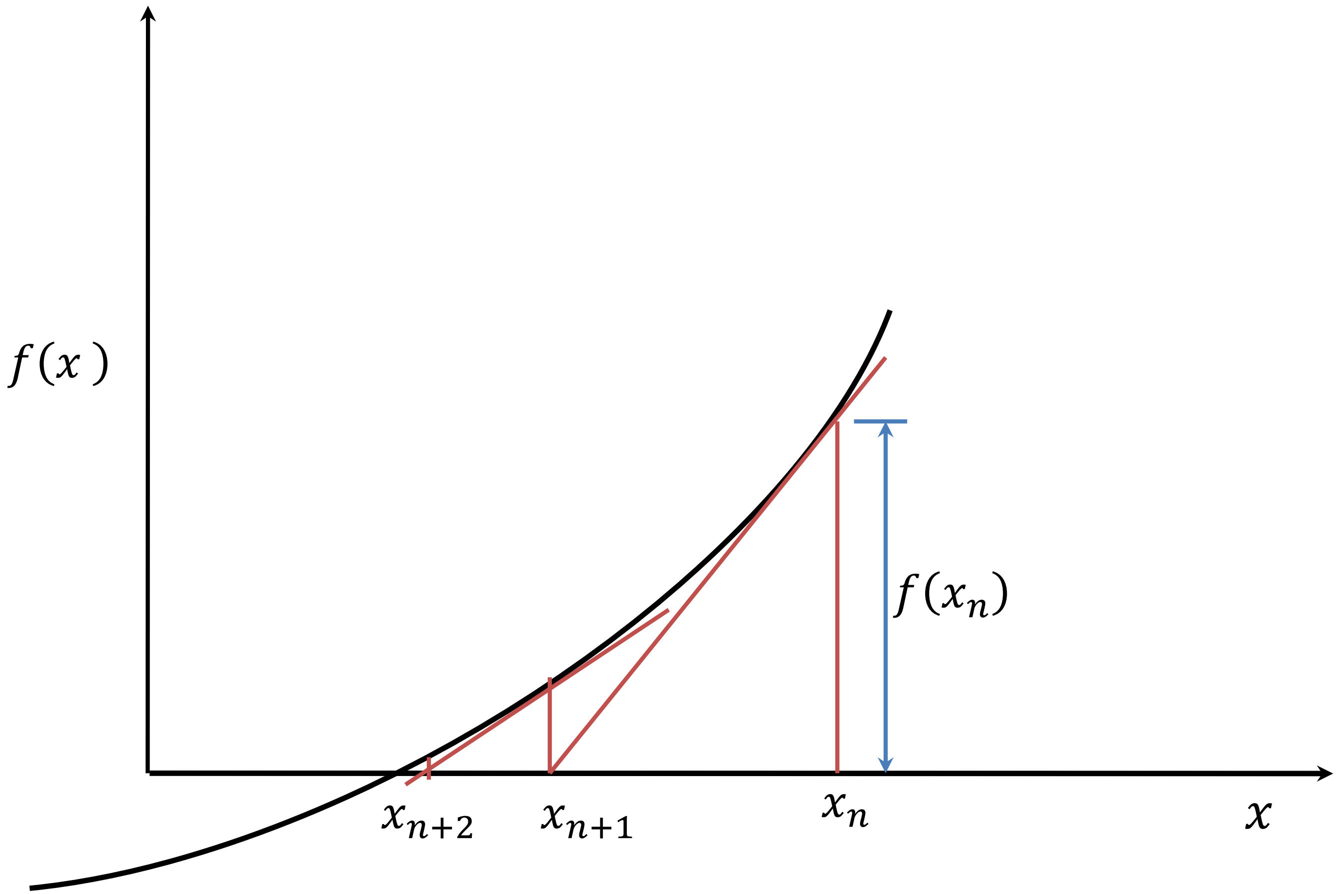
Search Method

Newton-Raphson method



$$f'(x_n) = \frac{f(x_n) - f(x_{n+1})}{x_n - x_{n+1}}$$

Rearranging and putting $f(x_{n+1}) = 0$

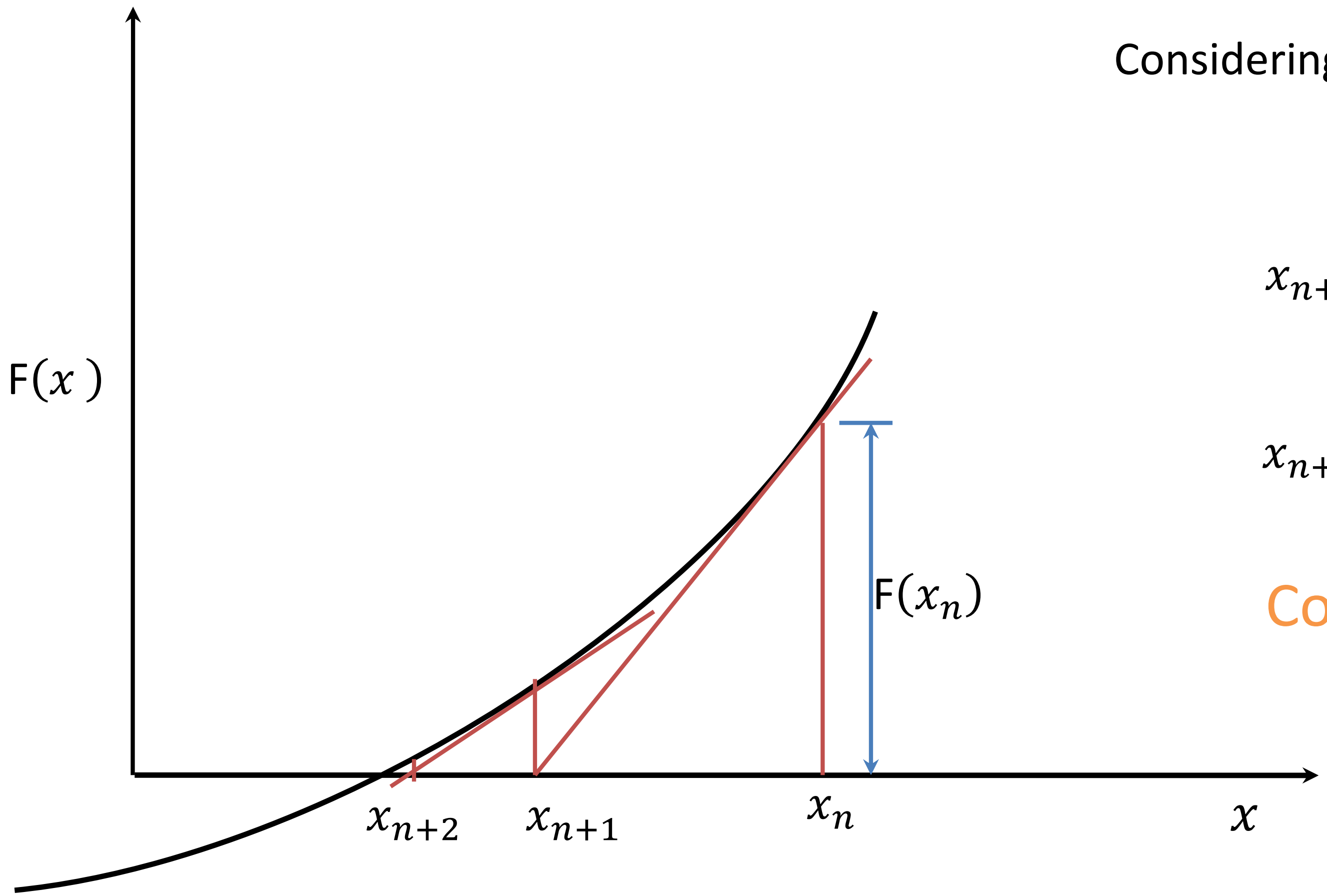
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+2} = x_{n+1} - \frac{f(x_{n+1})}{f'(x_{n+1})}$$

Continue iteration

Search Method

Newton-Raphson method



Incase optimization problem, $f'(x) = 0$

Considering $F(x) = f'(x)$

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

Continue iteration

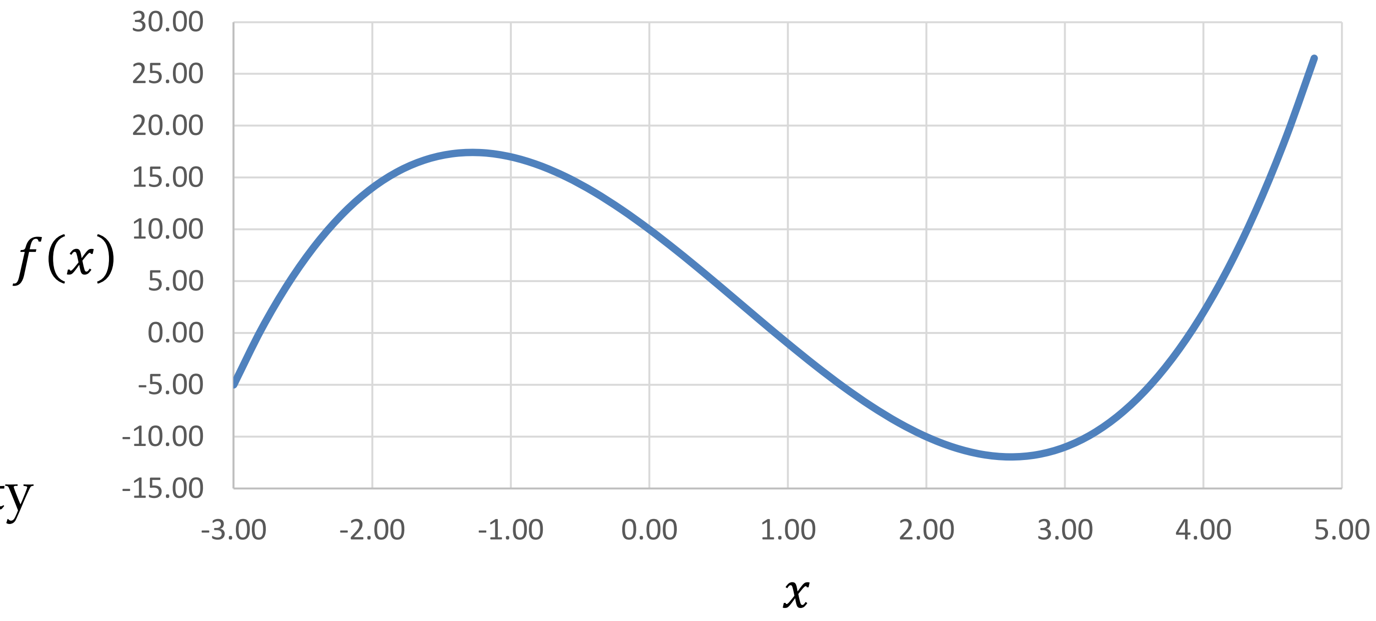
Search Method

An example

Example No. 1

$$f(x) = x^3 - 2x^2 - 10x - 10$$

$$f'(x) = 3x^2 - 4x - 10$$



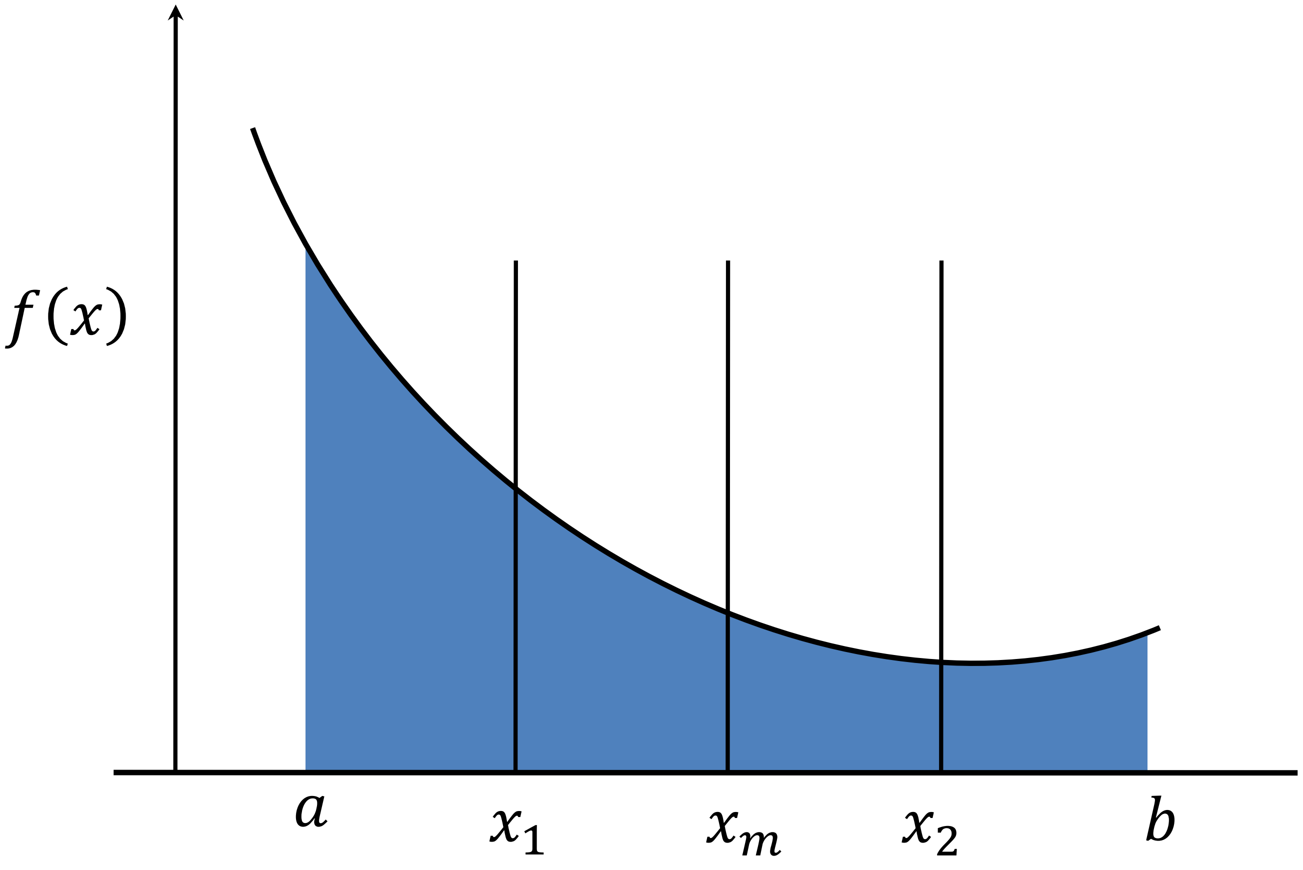
Necessary condition for optimality

$$f'(x) = 3x^2 - 4x - 10 = 0$$

Solving for x $\quad$ $\mathbf{x^* = 2.6103}$, **-1.277**

Search Method

Interval halving method



$if\ f(x_1) < f(x_m)$
eliminate $[x_m, x_2]$, i.e. $a = a, b = x_m$

$if\ f(x_2) < f(x_m)$
eliminate $[x_1, x_m]$, i.e. $a = x_m, b = b$

$if\ f(x_m) < f(x_1)\ \&\& f(x_m) < f(x_2)$
eliminate $[a, x_1]\ and [x_1, b]$
i.e. $a = x_1, b = x_2$

Search Method

Interval halving method

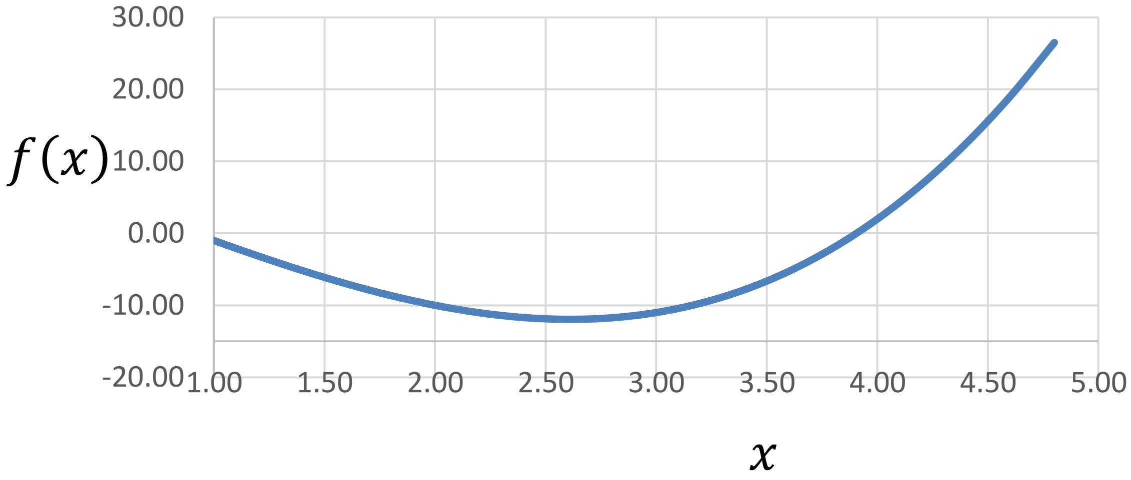
Example No. 1

$$f(x) = x^3 - 2x^2 - 10x - 10$$

Condition 1: $a = a, b = x_m$

Condition 2: $a = x_m, b = b$

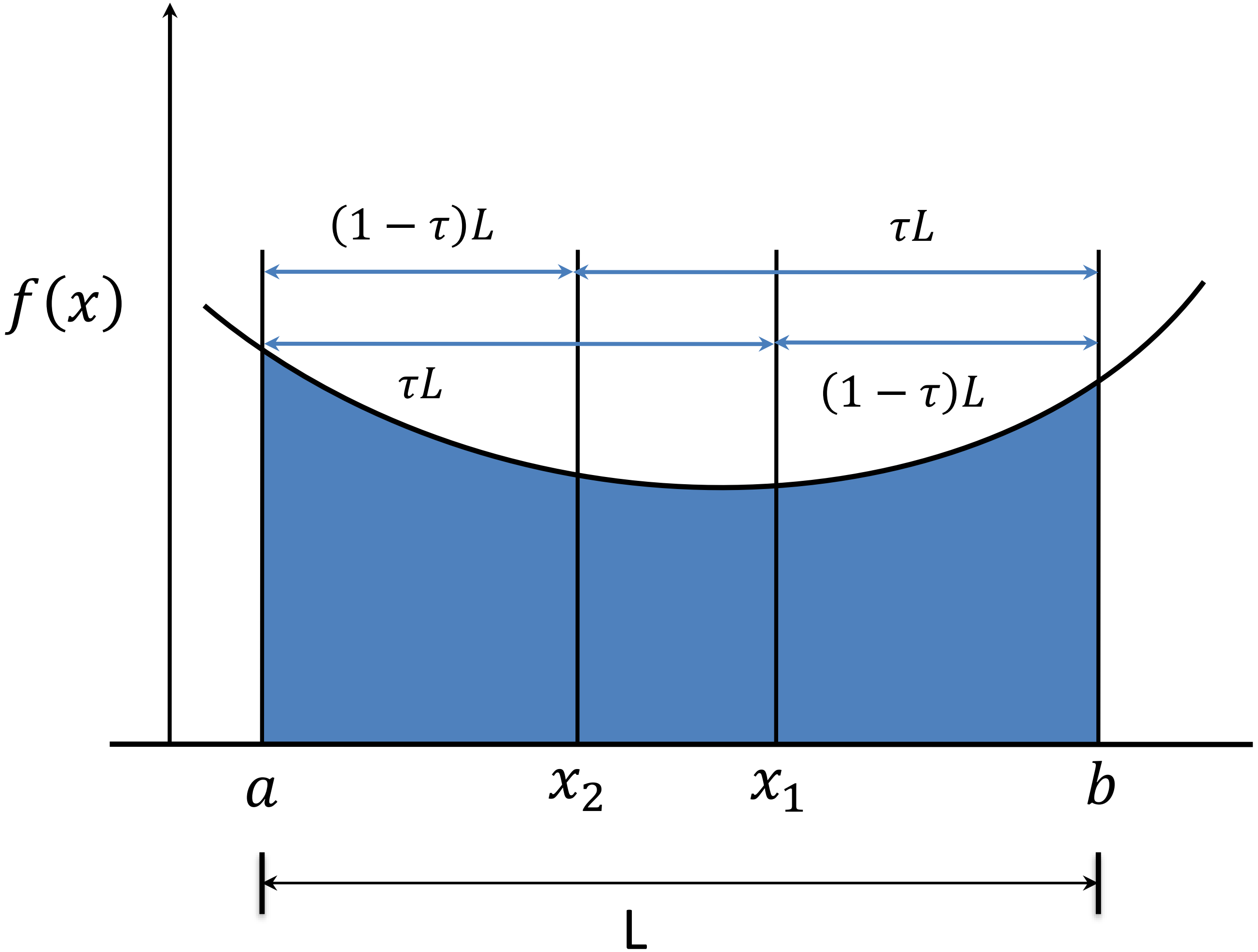
Condition 3: $a = x_1, b = x_2$



Iteration	a	b	x_1	x_m	x_2	$f(x_1)$	$f(x_m)$	$f(x_2)$	Condition	$\|a-b\|$	$\|a-b\|/2$
1	0	5	1.25	2.50	3.75	-3.671875000	-11.875000000	-2.890625000	$f(x_m) < f(x_1)$ &&$f(x_m) < f(x_2)$	5	2.5
2	1.25	3.75	1.88	2.50	3.13	-9.189453125	-11.875000000	-10.263671875	$f(x_m) < f(x_1)$ &&$f(x_m) < f(x_2)$	2.5	2.5
3	1.88	3.13	2.19	2.50	2.81	-10.977783203	-11.875000000	-11.697998047	$f(x_m) < f(x_1)$ &&$f(x_m) < f(x_2)$	1.25	2.5
4	2.19	2.81	2.34	2.50	2.66	-11.549224854	-11.875000000	-11.932220459	$f(x_2) < f(x_m)$	0.625	2.5
5	2.50	2.81	2.58	2.66	2.73	-11.938610077	-11.932220459	-11.852970123	$f(x_1) < f(x_m)$	0.3125	2.65625
6	2.50	2.66	2.54	2.58	2.62	-11.915376186	-11.938610077	-11.944344044	$f(x_2) < f(x_m)$	0.15625	2.578125
7	2.58	2.66	2.60	2.62	2.64	-11.943686903	-11.944344044	-11.940536797	$f(x_m) < f(x_1)$ &&$f(x_m) < f(x_2)$	0.078125	2.6171875
8	2.60	2.64	2.61	2.62	2.63	-11.944570728	-11.944344044	-11.943001263	$f(x_1) < f(x_m)$	0.0390625	2.6171875
9	2.60	2.62	2.60	2.61	2.61	-11.944267279	-11.944570728	-11.944596549	$f(x_2) < f(x_m)$	0.01953125	2.607421875
10	2.61	2.62	2.61	2.61	2.61	-11.944618385	-11.944596549	-11.944505131	$f(x_1) < f(x_m)$	0.009765625	2.612304688
11	2.61	2.61	2.61	2.61	2.61	-11.944603238	-11.944618385	-11.944616159	$f(x_m) < f(x_1)$ &&$f(x_m) < f(x_2)$	0.004882813	2.609863281
12	2.61	2.61	2.61	2.61	2.61	-11.944612982	-11.944618385	-11.944619445	$f(x_2) < f(x_m)$	0.002441406	2.609863281
13	2.61	2.61	2.61	2.61	2.61	-11.944619458	-11.944619445	-11.944618345		0.001220703	**2.610473633**

Search Method

Golden Section Method



Apply region elimination rules

Suppose

If $f(x_1) > f(x_2)$
then $a = a, b = x_1$

If $f(x_2) > f(x_1)$
then $a = x_2, b = b$

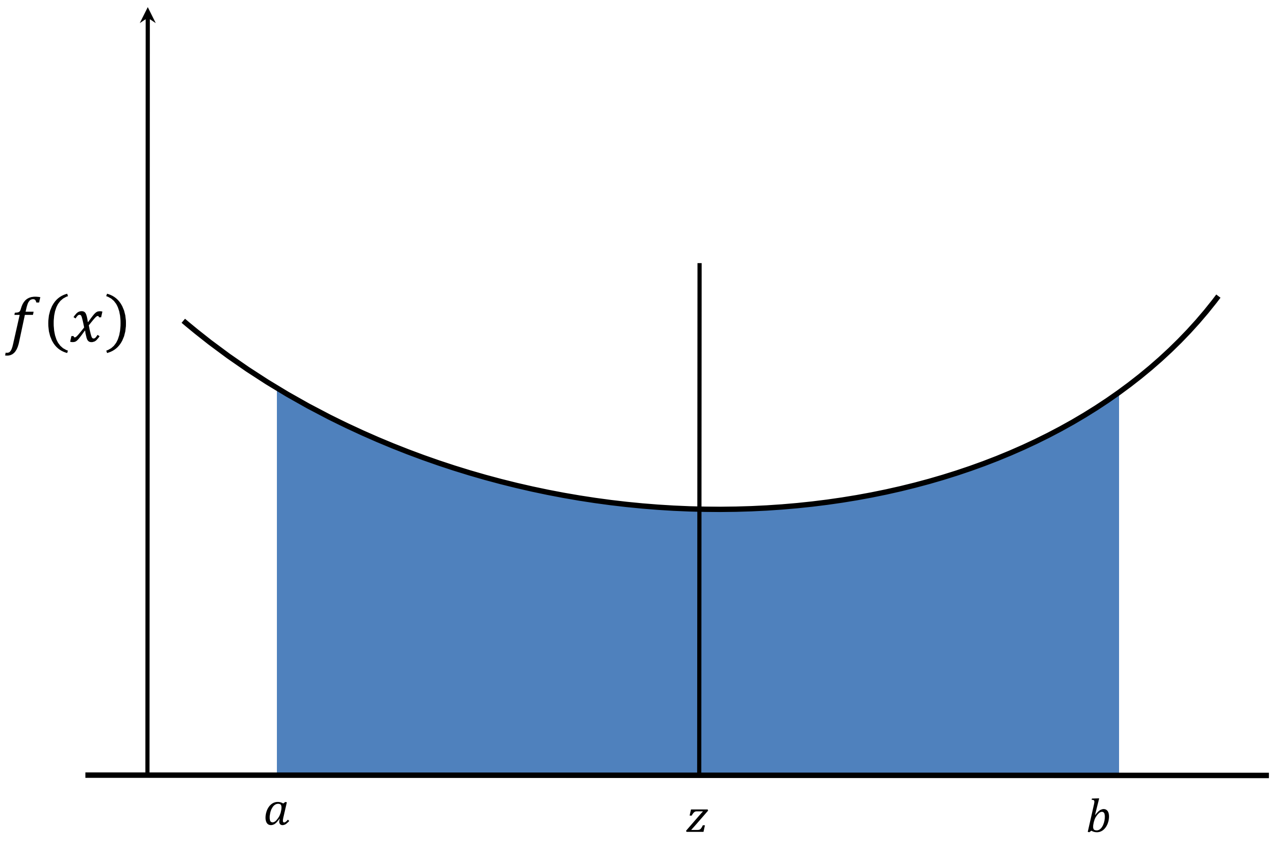
Search Method

Golden Section Method

Iteration	a	b	x_1	x_2	$f(x_1)$	$f(x_2)$	Condition	$\|a-b\|$	$\|a-b\|/2$
1	0.000000	5.000000	3.090000	1.910000	-10.49257100	-9.42832900	f2>f1	5	2.5
2	1.910000	5.000000	3.819620	3.090380	-1.64885957	-10.49018192	f1>f2	3.09	3.455
3	1.910000	3.819620	3.090145	2.639475	-10.49165862	-11.93963754	f1>f2	1.90962000000000000000	2.86481
4	1.910000	3.090145	2.639330	2.360815	-11.93968714	-11.59716778	f2>f1	1.18014516000000000000	2.50007258
5	2.360815	3.090145	2.811541	2.639419	-11.70037038	-11.93965652	f1>f2	0.72932970888000000000	2.72548031
6	2.360815	2.811541	2.639364	2.532993	-11.93967545	-11.91021810	f2>f1	0.45072576008784000000	2.58617833
7	2.532993	2.811541	2.705136	2.639398	-11.89134380	-11.93966376	f1>f2	0.27854851973428500000	2.67226695
8	2.532993	2.705136	2.639377	2.598751	-11.93967099	-11.94384112	f1>f2	0.17214298519578800000	2.61906418
9	2.532993	2.639377	2.598738	2.573632	-11.94383936	-11.93682139	f2>f1	0.10638436485099700000	2.58618487
10	2.573632	2.639377	2.614262	2.598746	-11.94452878	-11.94384044	f2>f1	0.06574553747791610000	2.60650429
11	2.598746	2.639377	2.623856	2.614267	-11.94354829	-11.94452855	f1>f2	0.04063074216135210000	2.61906169
12	2.598746	2.623856	2.614264	2.608338	-11.94452869	-11.94459676	f1>f2	0.02510979865571540000	2.61130121
13	2.598746	2.614264	2.608336	2.604674	-11.94459671	-11.94443408	f2>f1	0.01551785556923190000	2.60650524
14	2.604674	2.614264	2.610601	2.608338	-11.94461912	-11.94459674	f2>f1	0.00959003474178521000	2.60946915
15	2.608338	2.614264	2.612000	2.610602	-11.94460307	-11.94461912	f1>f2	0.00592664147042310000	2.61130085
16	2.608338	2.612000	2.610601	2.609737	-11.94461912	-11.94461762	f2>f1	0.00366266442872165000	2.61016886
17	2.609737	2.612000	2.611136	2.610601	-11.94461568	-11.94461912	f1>f2	0.00226352661695017000	2.61086843
18	2.609737	2.611136	2.610601	2.610271	-11.94461912	-11.94461957	f1>f2	0.00139885944927531000	2.6104361
19	2.609737	2.610601	2.610271	2.610067	-11.94461957	-11.94461922	f2>f1	0.00086449513965236600	2.61016891
20	2.610067	2.610601	2.610397	2.610271	-11.94461955	-11.94461957	f1>f2	0.00053425799630524300	2.61033403

Search Method

Bisection method



Take a point $z = \frac{a+b}{2}$

$if\, f'(z) < 0$ then area between $[a, z]$ will be eliminated

$if\, f'(z) > 0$ then area between $[z, b]$ will be eliminated

Search Method

Bisection method

Iteration No	a	b	x_m	$f(x_m)$	Condition	$\|a-b\|$	$\|a-b\|/2$
1	0.00000	5.00000	2.50000	-1.250000000	Negative	5	2.5
2	2.50000	5.00000	3.75000	17.187500000	Positive	2.5	3.75
3	2.50000	3.75000	3.12500	6.796875000	Positive	1.25	3.125
4	2.50000	3.12500	2.81250	2.480468750	Positive	0.625	2.8125
5	2.50000	2.81250	2.65625	0.541992188	Positive	0.3125	2.65625
6	2.50000	2.65625	2.57813	-0.372314453	Negative	0.15625	2.578125
7	2.57813	2.65625	2.61719	0.080261230	Positive	0.078125	2.6171875
8	2.57813	2.61719	2.59766	-0.147171021	Negative	0.0390625	2.59765625
9	2.59766	2.61719	2.60742	-0.033740997	Negative	0.01953125	2.607421875
10	2.60742	2.61719	2.61230	0.023188591	Positive	0.00976563	2.612304688
11	2.60742	2.61230	2.60986	-0.005294085	Negative	0.00488281	2.609863281
12	2.60986	2.61230	2.61108	0.008942783	Positive	0.00244141	2.611083984
13	2.60986	2.61108	2.61047	0.001823232	Positive	0.0012207	2.610473633
14	2.60986	2.61047	2.61017	-0.001735706	Negative	0.00061035	2.610168457
15	2.61017	2.61047	2.61032	0.000043693	Positive	0.00030518	2.610321045
16	2.61017	2.61032	2.61024	-0.000846024	Negative	0.00015259	2.610244751
17	2.61024	2.61032	2.61028	-0.000401170	Negative	7.6294E-05	2.610282898
18	2.61028	2.61032	2.61030	-0.000178740	Negative	3.8147E-05	2.610301971
19	2.61030	2.61032	2.61031	-0.000067524	Negative	1.9073E-05	2.610311508
20	2.61031	2.61032	2.61032	-0.000011915	Negative	9.5367E-06	2.610316277

Search Method

Newton-Raphson method

Example No. 1

$$f(x) = x^3 - 2x^2 - 10x - 10$$

$$f'(x) = 3x^2 - 4x - 10$$

$$f''(x) = 6x - 4$$

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

Solution

$x_0 = 1$

Iteration	x_n	$f'(x)$	$f''(x)$	x_{n+1}
1	1	-11	2	6.5
2	6.5	90.75	35	3.9071
3	3.9071	20.169	19.443	2.8698
4	2.8698	3.2282	13.219	2.6256
5	2.6256	0.1789	11.754	2.6104
6	2.6104	0.0007	11.662	2.6103
7	2.6103	1E-08	11.662	**2.6103**

$x_0 = 0$

Iteration	x_n	$f'(x)$	$f''(x)$	x_{n+1}
1	0	-10	-4	-2.5
2	-2.5	18.75	-19	-1.513
3	-1.513	2.9216	-13.08	-1.29
4	-1.29	0.1497	-11.74	-1.277
5	-1.277	0.0005	-11.66	-1.277
6	-1.277	5E-09	-11.66	-1.277
7	-1.277	0	-11.66	**-1.277**

Search Method

Thanks