Quadratic approximation

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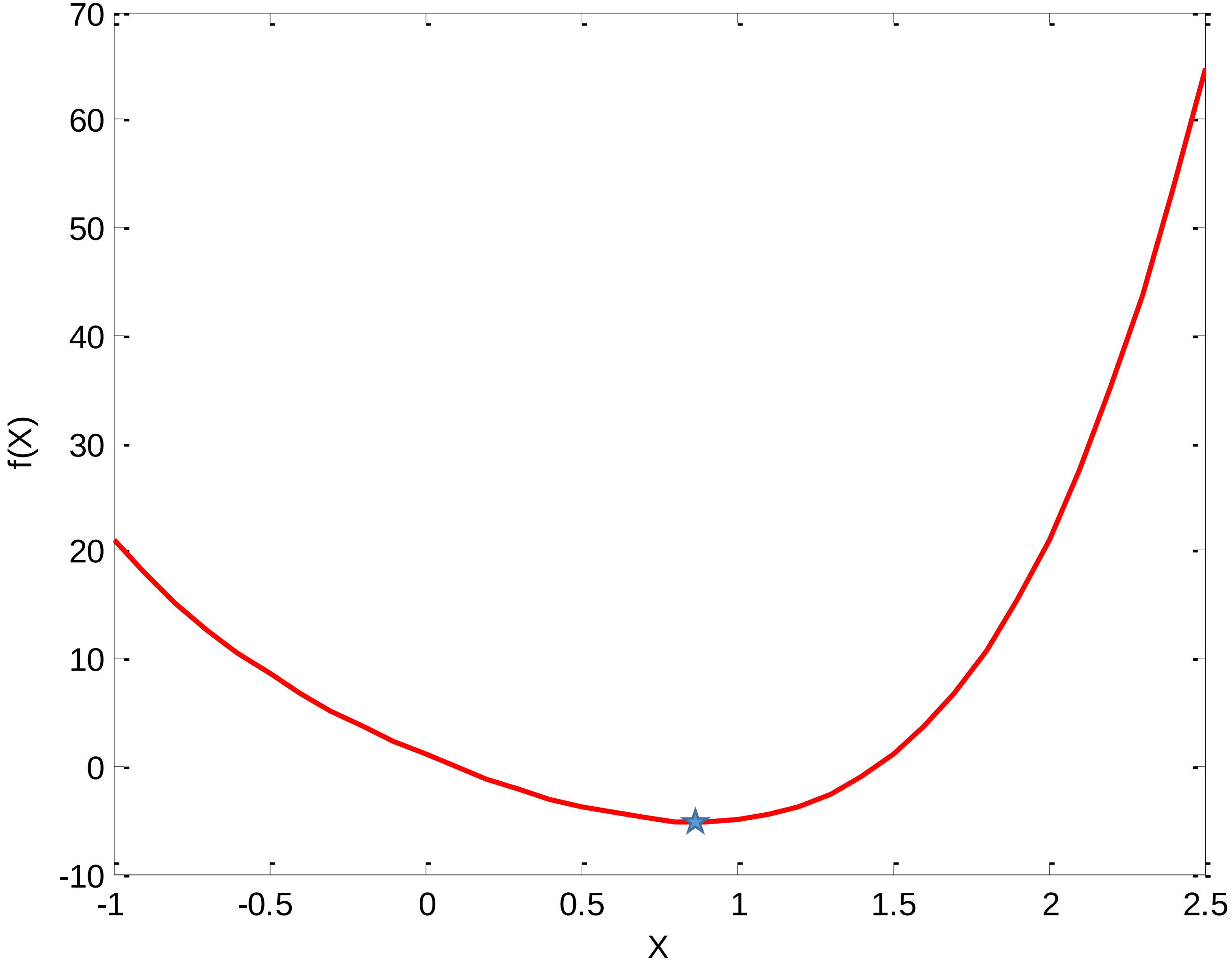
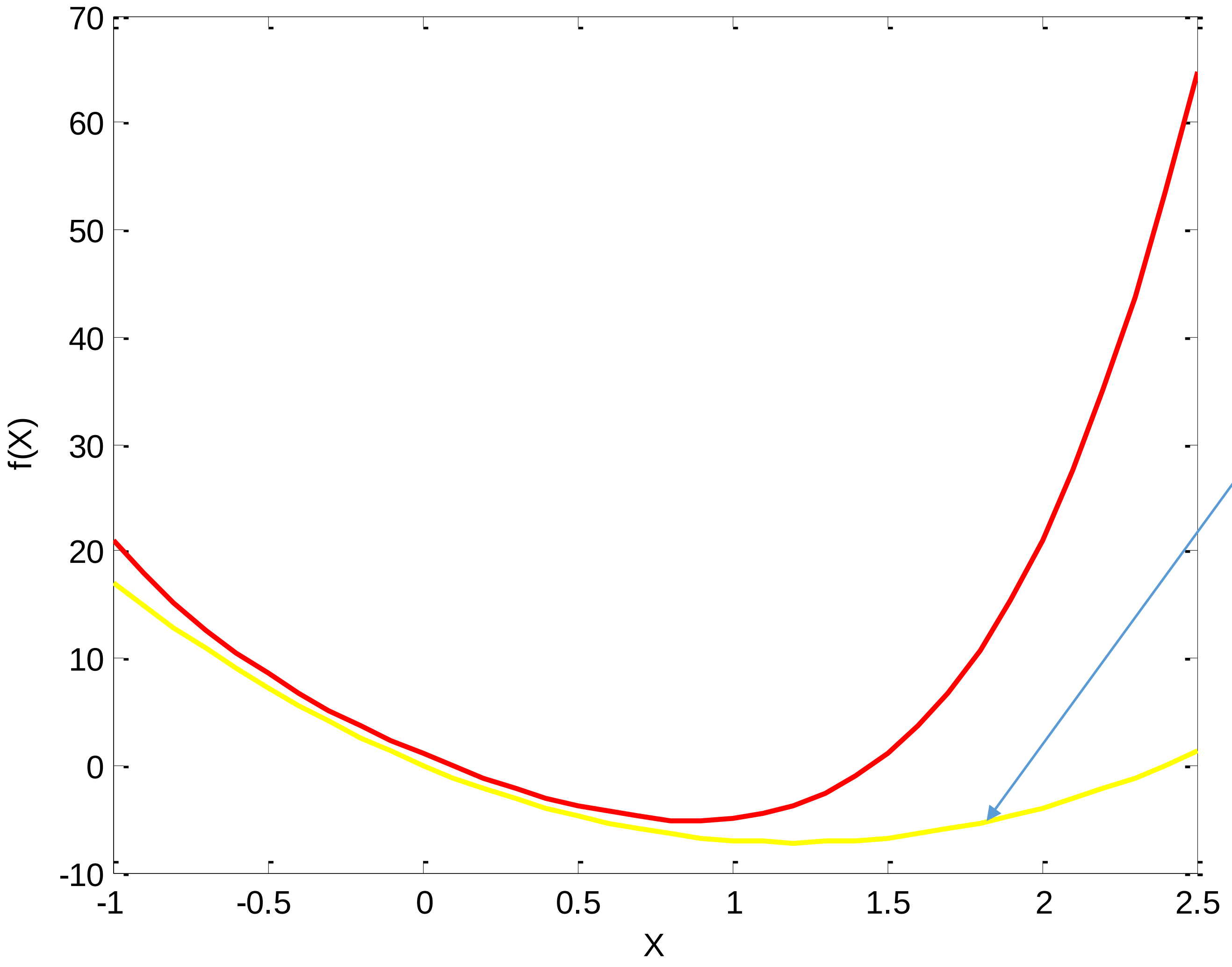


$$f(x) = 2x^4 - x^3 + 5x^2 - 12x + 1$$

$$f^{/}(x) = 8x^3 - 3x^2 + 10x - 12 = 0$$

Solving for x

$x^* = 0.8831$ and $f(x^*) = -5.1702$



$f(x) = 2x^4 - x^3 + 5x^2 - 12x + 1$

Quadratic approximation of the function at x_o can be written as

$$f(x) = f(x_o) + f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$$

Approximate function for $x_o = 0$

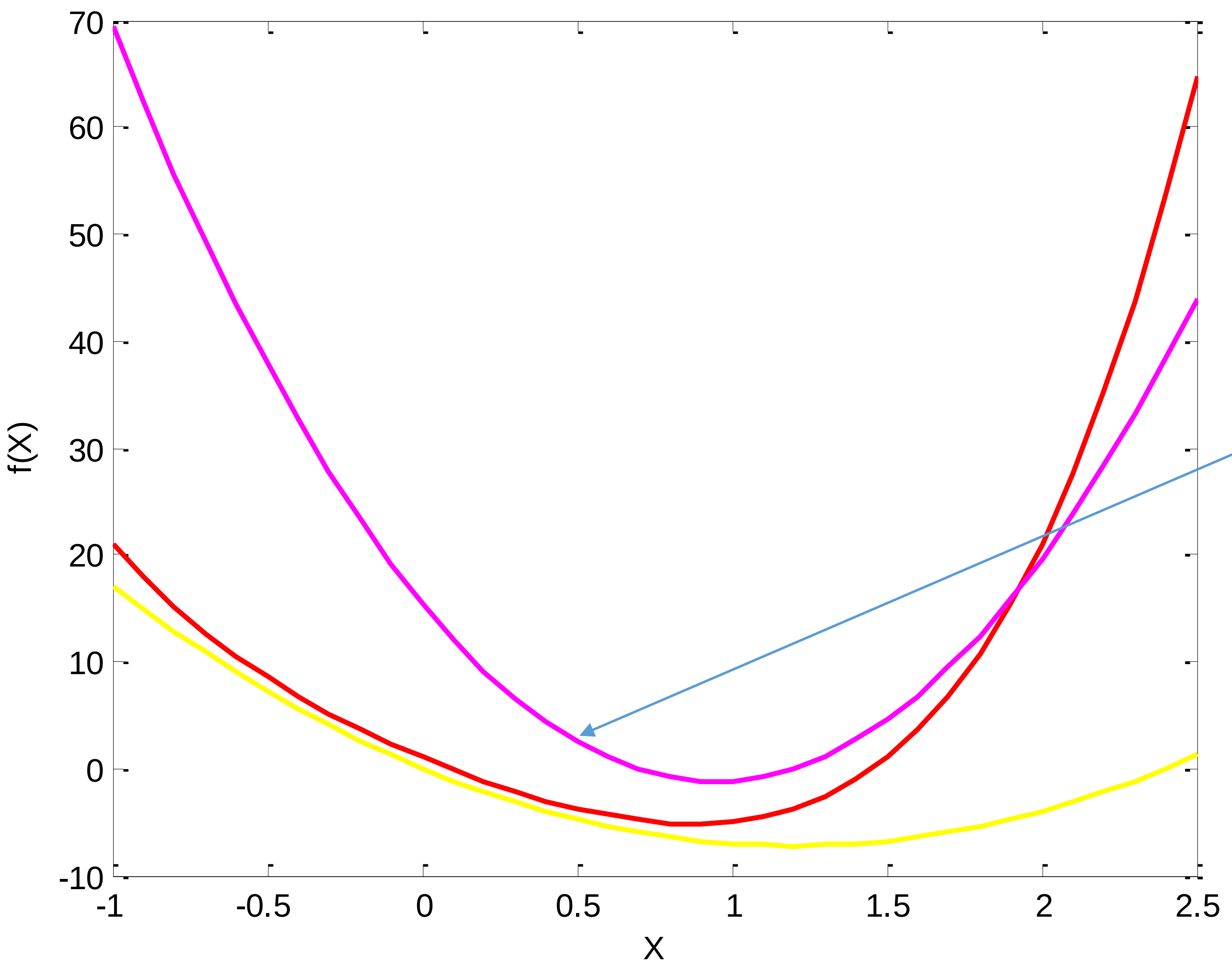
Now we can minimize the function

Minimize $f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$

Solution is

$x^* = 1.2$ and $f(x^*) = -3.7808$ and $f^{/}(x^*) = 9.5040$

This is the solution of the approximate function: First trial



$$f(x) = 2x^4 - x^3 + 5x^2 - 12x + 1$$

Quadratic approximation of the function at x_o can be written as

$$f(x) = f(x_o) + f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$$

Approximate function for $x_o = 1.2$

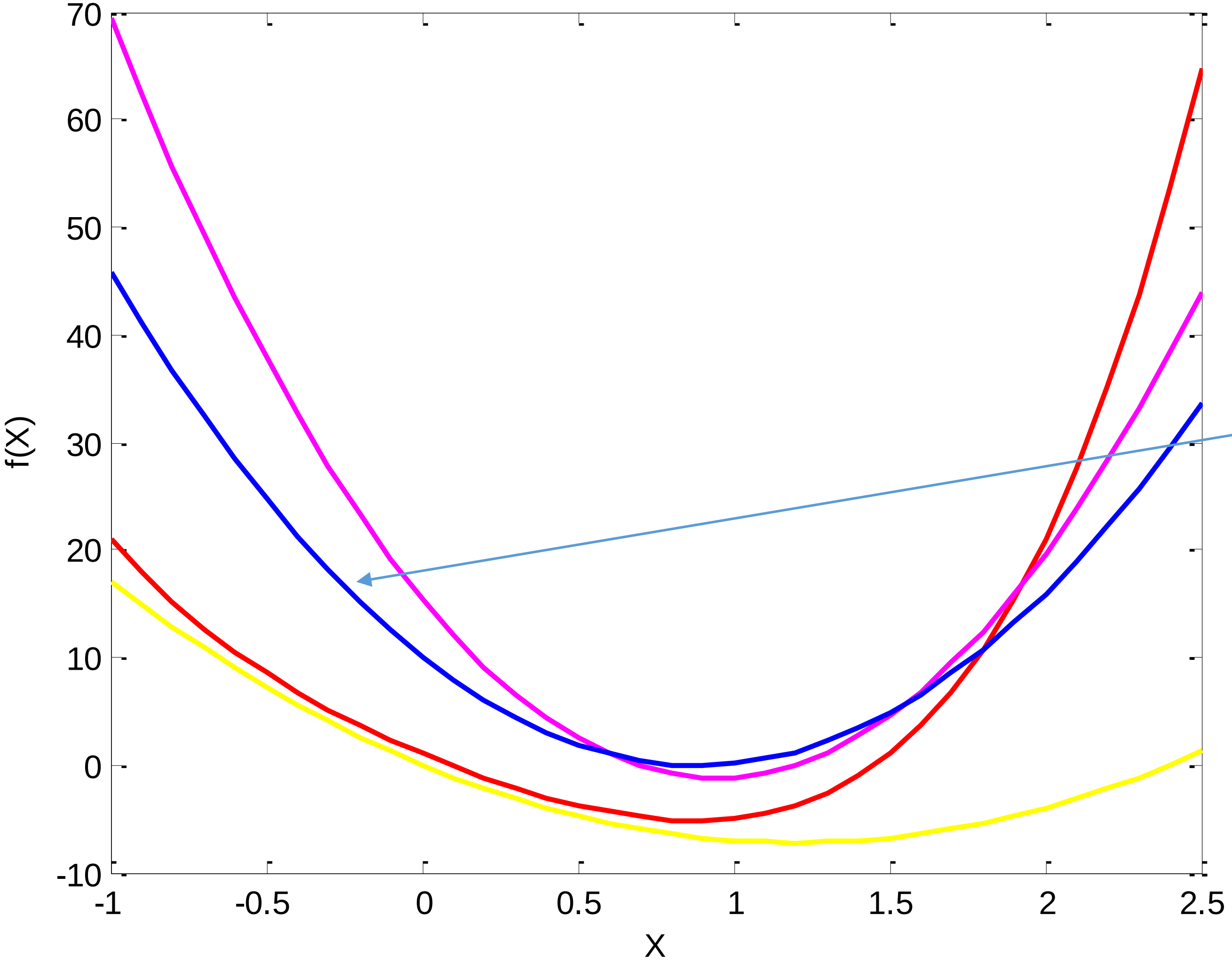
Now we can minimize the function

Minimize $f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$

Solution is

$x^* = 0.9456, f(x^*) = -5.1229$ and $f^{/}(x^*) = 1.5377$

This is the solution of the approximate function: Second trial



$$f(x) = 2x^4 - x^3 + 5x^2 - 12x + 1$$

Quadratic approximation of the function at x_o can be written as

$$f(x) = f(x_o) + f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$$

Approximate function for $x_o = 0.9456$

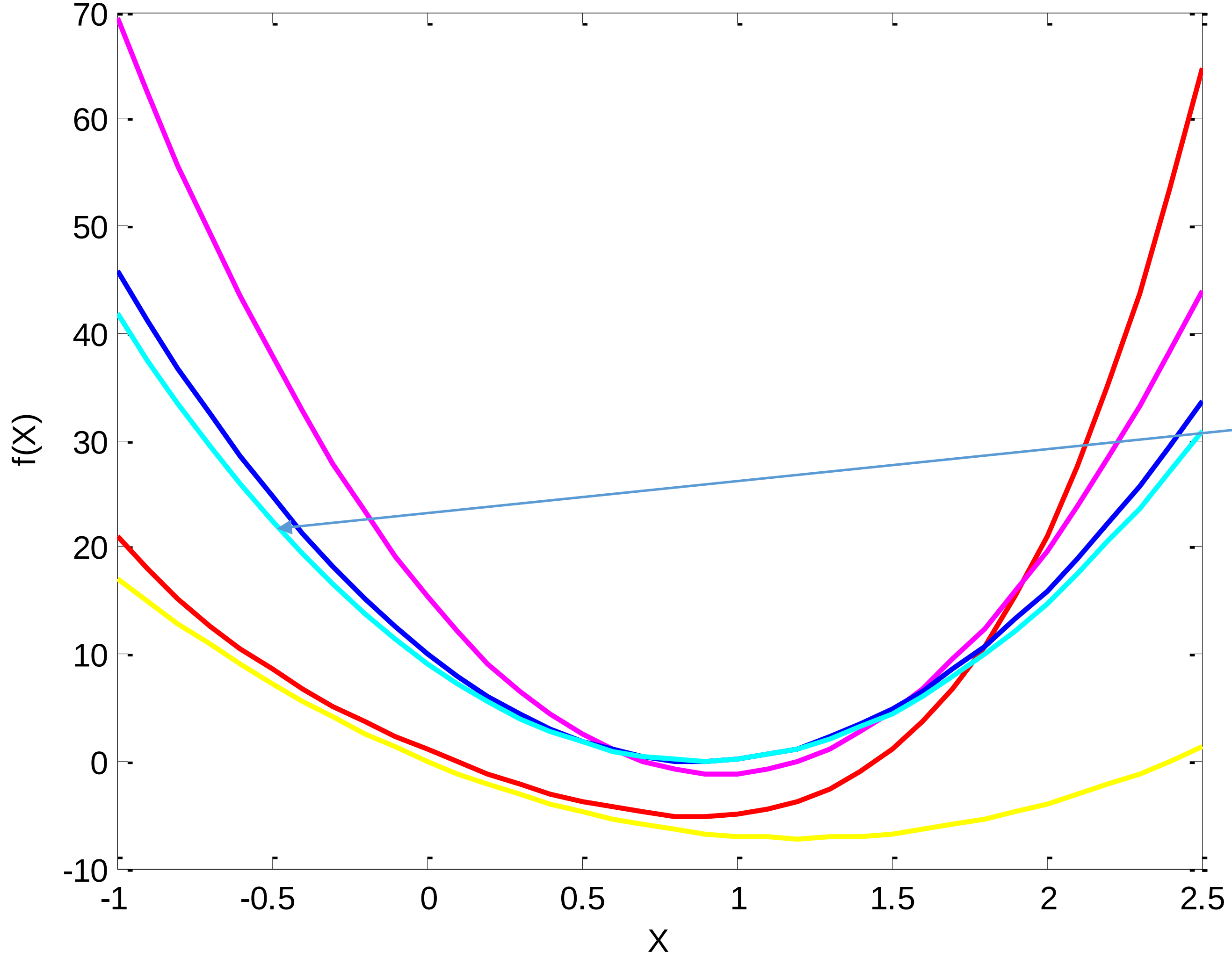
Now we can minimize the function

Minimize $f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$

Solution is

$x^* = 0.8864$ and $f(x^*) = -5.1701$ and $f^{/}(x^*) = 0.0785$

This is the solution of the approximate function: Third trial



$f(x) = 2x^4 - x^3 + 5x^2 - 12x + 1$

Quadratic approximation of the function at x_o can be written as

$$f(x) = f(x_o) + f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$$

Approximate function for $x_o = 0.8864$

Now we can minimize the function

Minimize $f^{/}(x_o)(x - x_o) + 0.5 * f^{//}(x_o)(x - x_o)^2$

Solution is

$x^* = 0.8831$ and $f(x^*) = -5.1702$ and $f^{/}(x^*) = 0.00099$

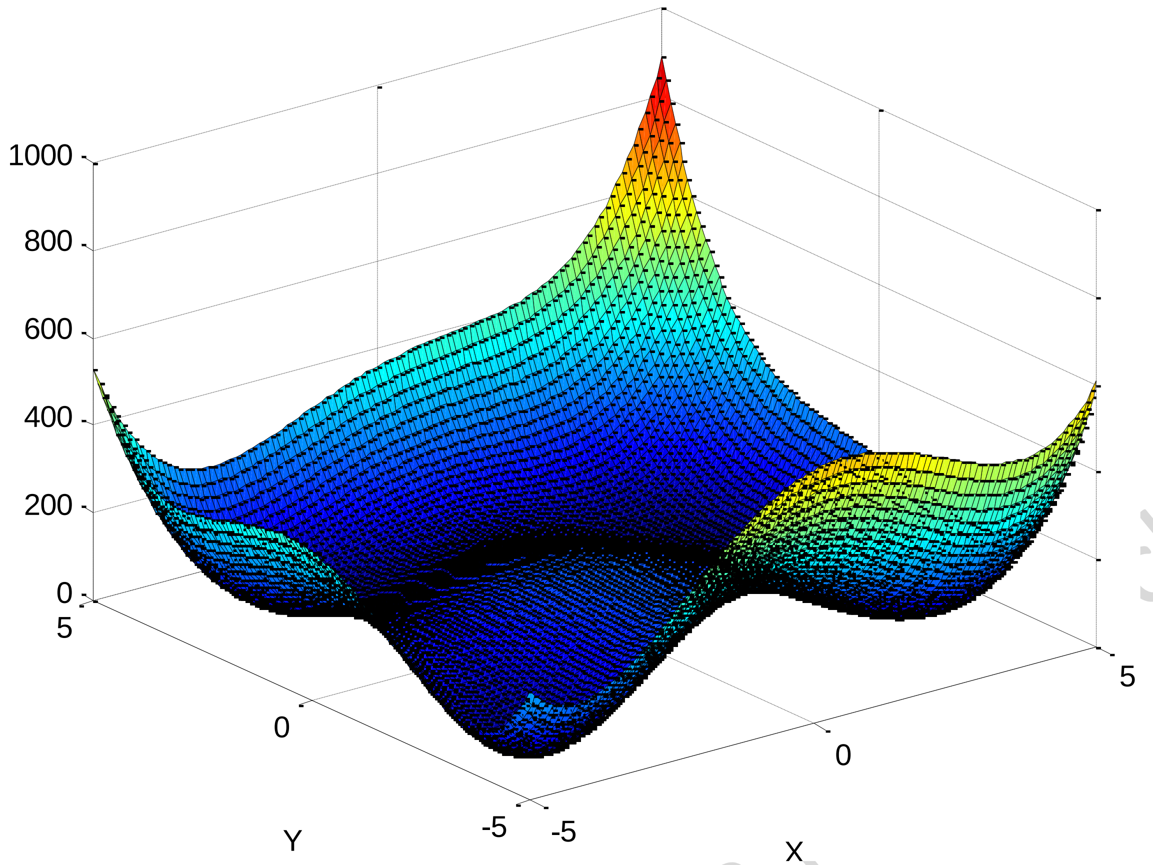
This is the solution of the approximate function: Fourth trial

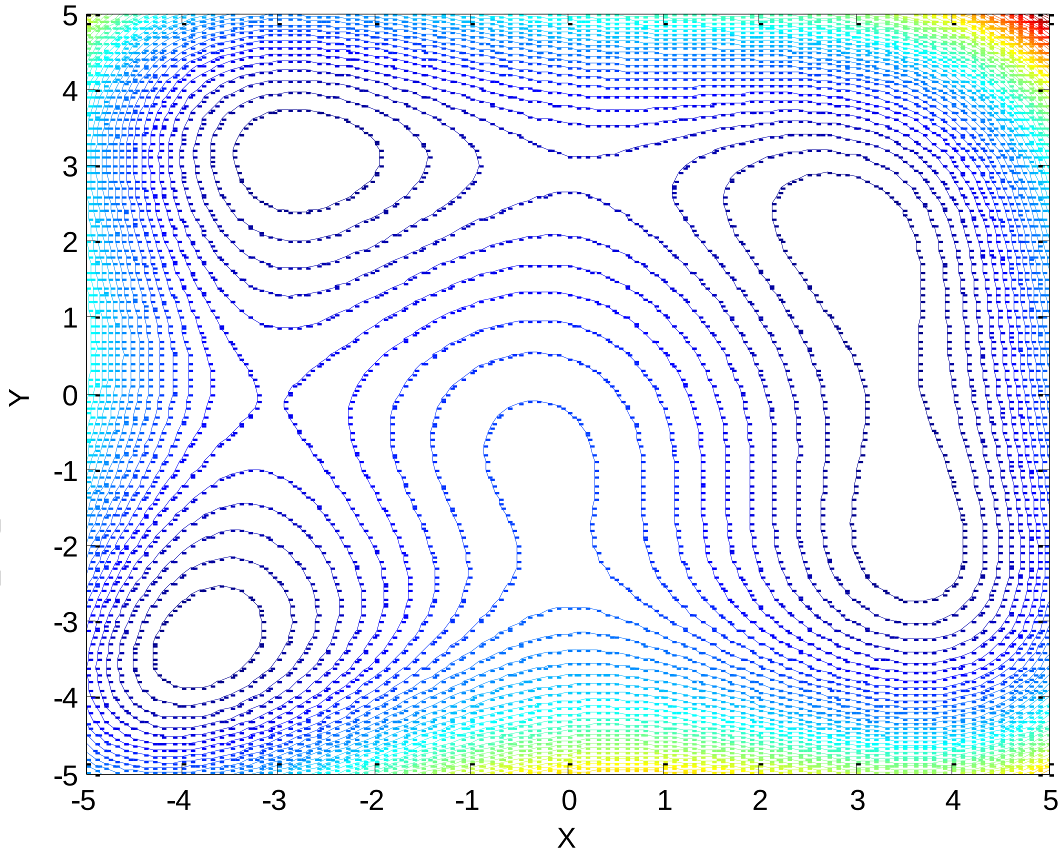
Gradient is negligible

STOP ITERATION

Now take an example of multivariable problem

Minimize $f(x_1, x_2) = (x_1^2 + x_2 - 11)^{\,2} + (x_1 + x_2^2 - 7)^{\,2}$





Minimize $f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$ $\quad x_o = [2\ 2]^T$

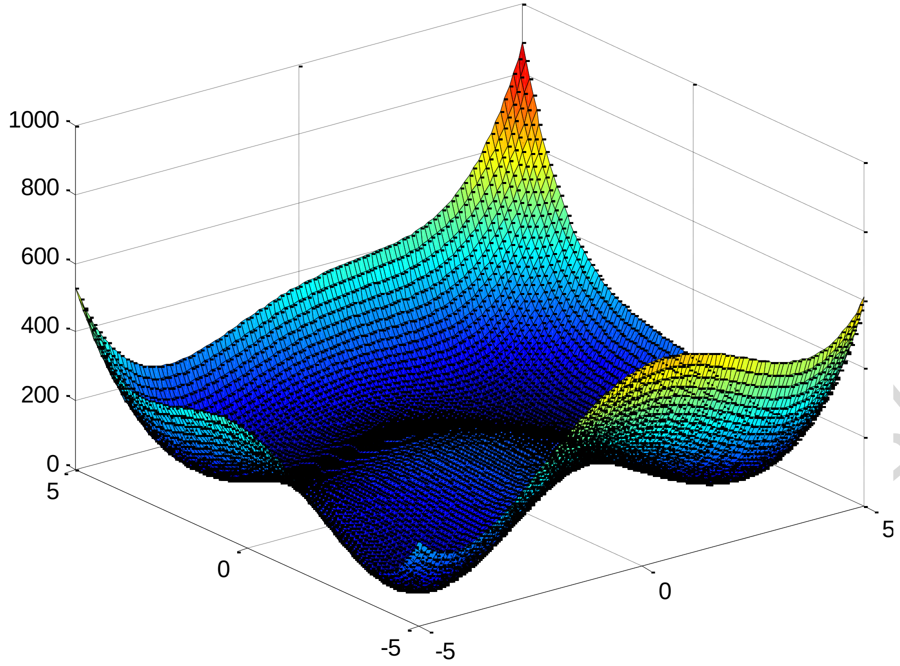
The quadratic approximation of the function at $x_o = [2\ 2]^T$ can be written as

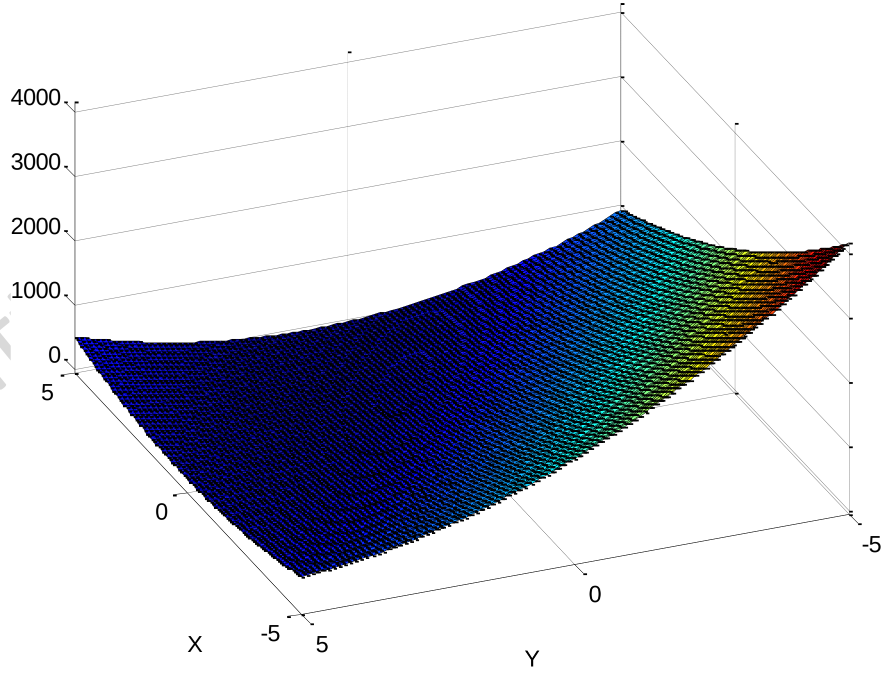
$$f(X) = f(X_o) + (X - X_o)\nabla f(X_o)^T + \frac{1}{2}(X - X_o)H(X_o)(X - X_o)^T$$

For first approximation

Minimize $$f(X) = (X - X_o)\nabla f(X_o)^T + \frac{1}{2}(X - X_o)H(X_o)(X - X_o)^T$$

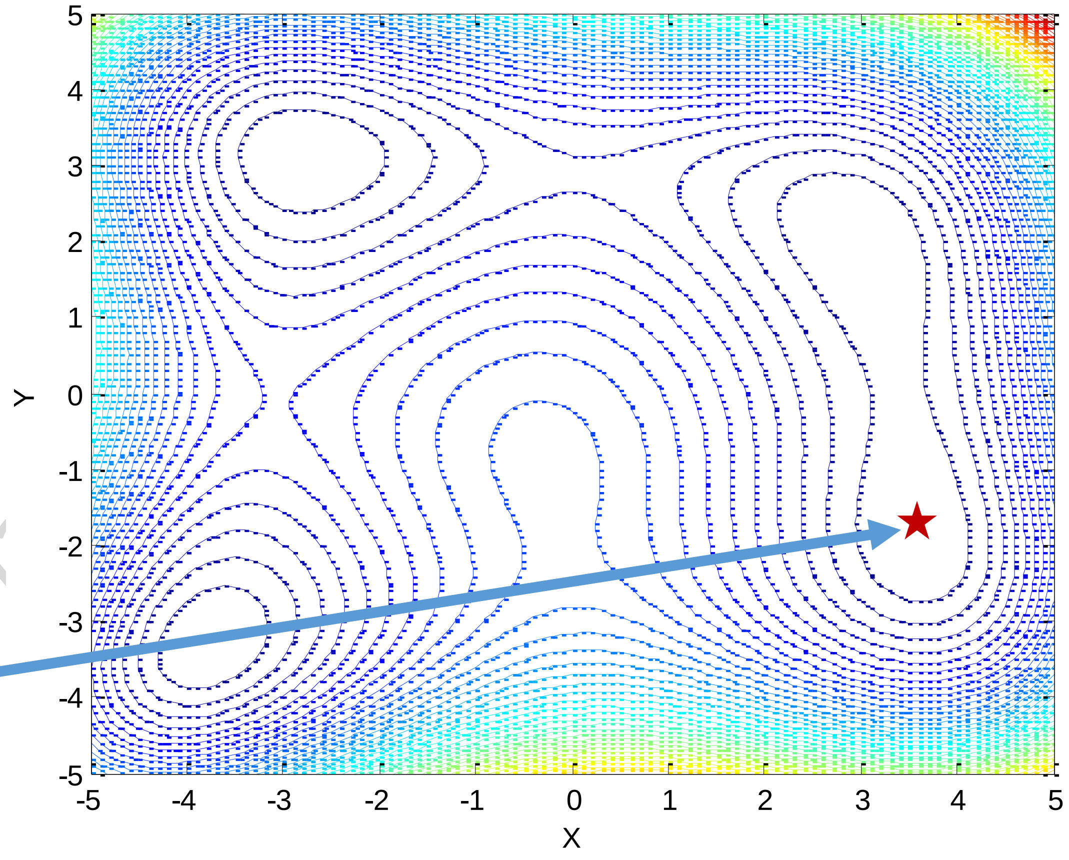
Or, $$f(X) = [x_1 - 2 \quad x_2 - 2]\begin{bmatrix} -42 \\ -18 \end{bmatrix} + \frac{1}{2}[x_1 - 2 \quad x_2 - 2]\begin{bmatrix} 14 & 16 \\ 16 & 30 \end{bmatrix}\begin{bmatrix} x_1 - 2 \\ x_2 - 2 \end{bmatrix}$$





Solution

Trial	X value		Gradient	
1	7.9268	-0.5610	-42	-18
2	5.7945	-4.4555	1628	99
3	4.4415	-3.1670	457	-296
4	3.7952	-2.3927	113	-83
5	3.6086	-1.9928	20	-22
6	3.5858	-1.8623	1.5811	-4.5637
7	3.5844	-1.8483	0.0457	-0.4106
8	3.5844	-1.8481	-0.0042	-0.0053
9	3.5844	-1.8481	-0.0028	0.0006



Optimal solution

Gradient is almost negligible

Thanks