Constrained Optimization

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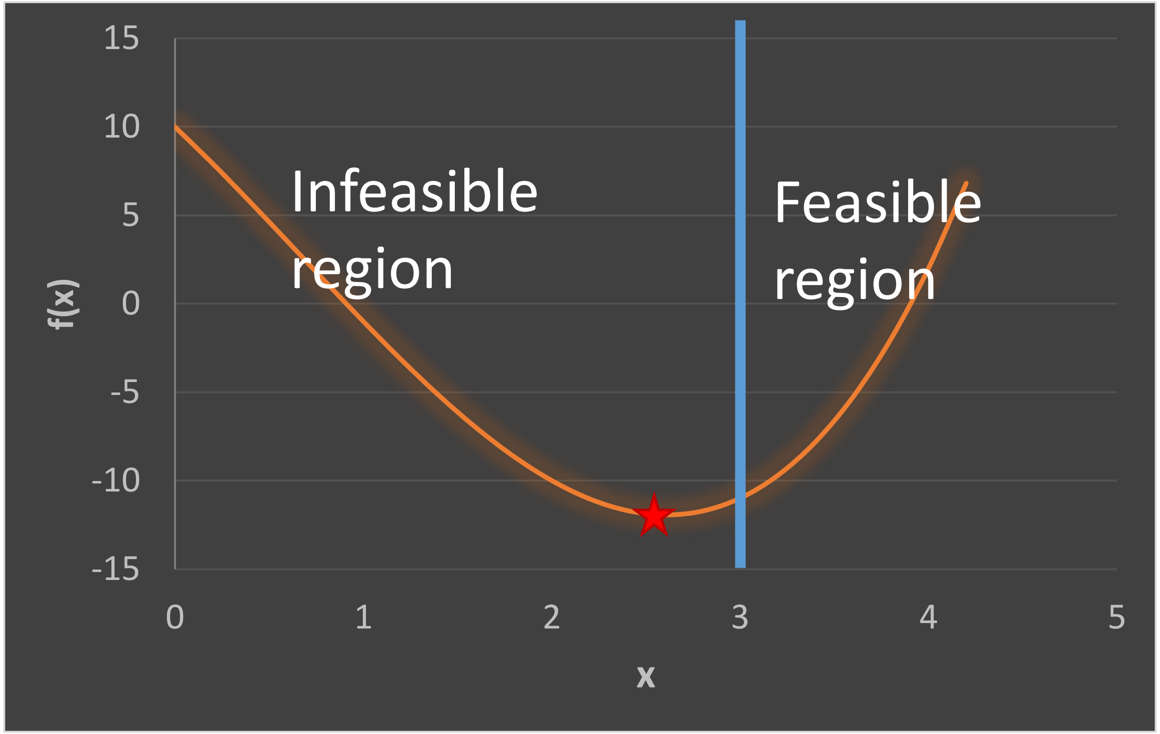
Constrained Optimization



Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

The problem can be written as

$$\mathrm{F}(x, R) = f(x) + R\langle g(x)\rangle^2$$

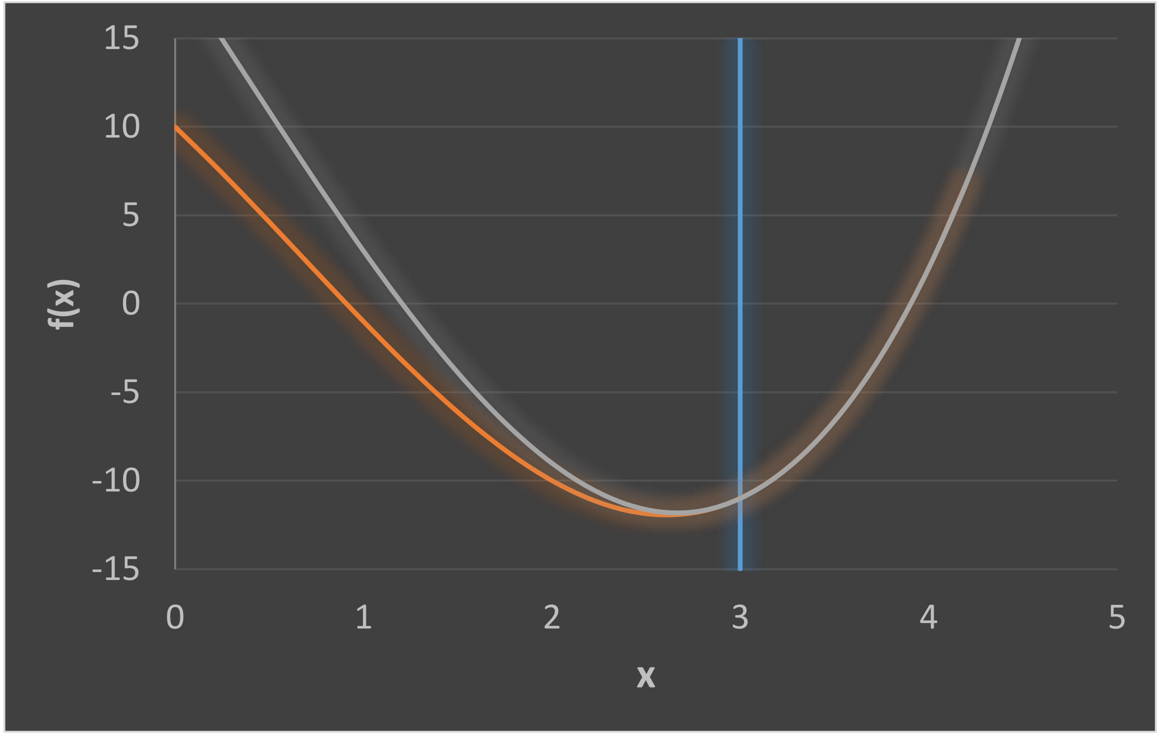
Where,

$\langle g(x)\rangle = 0$ if $x \geq 3$

$\langle g(x)\rangle = g(x)$ otherwise

The bracket operator $\langle \quad \rangle$ can be implemented using $min(g, 0)$ function

Constrained Optimization



Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

The problem can be written as

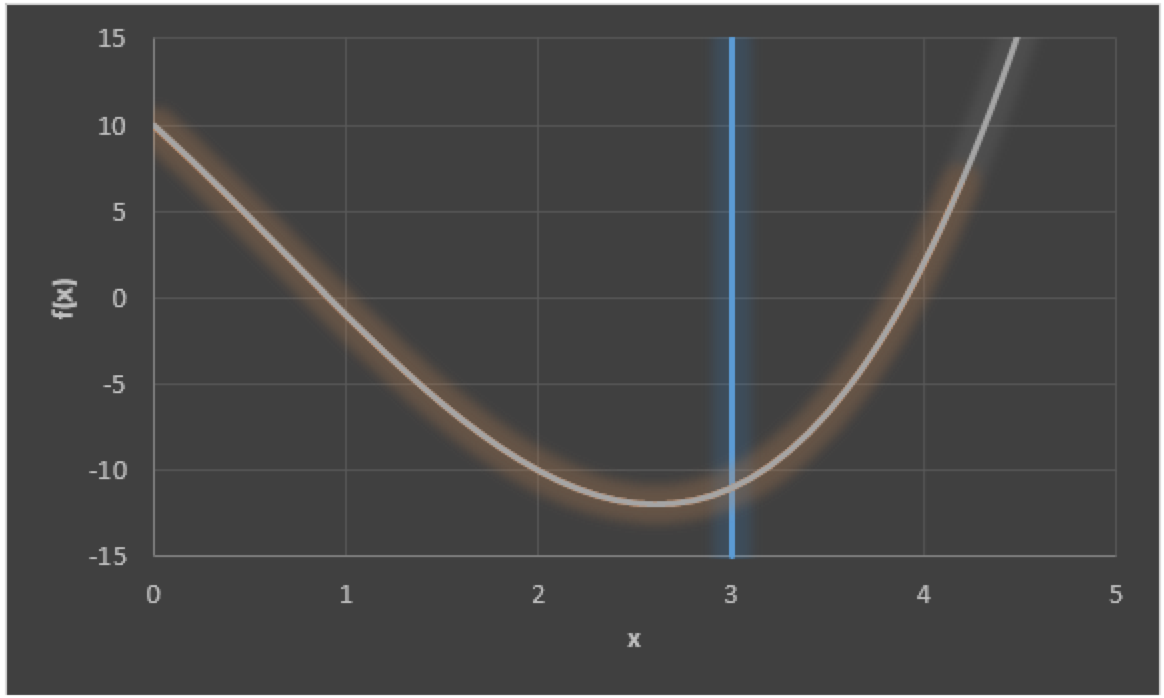
$$\mathrm{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\langle x - 3\rangle^2$$

$$\mathrm{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\big(min(x - 3,0)\big)^2$$

Constrained Optimization

Minimize $F(x, R) = (x^3 - 10x - 2x^2 + 10) + R\left(min(x - 3,0)\right)^2$

R 0



By changing R value, it is possible to avoid the infeasible solution

The minimization of the transformed function will provide the optimal solution which is in the feasible region only

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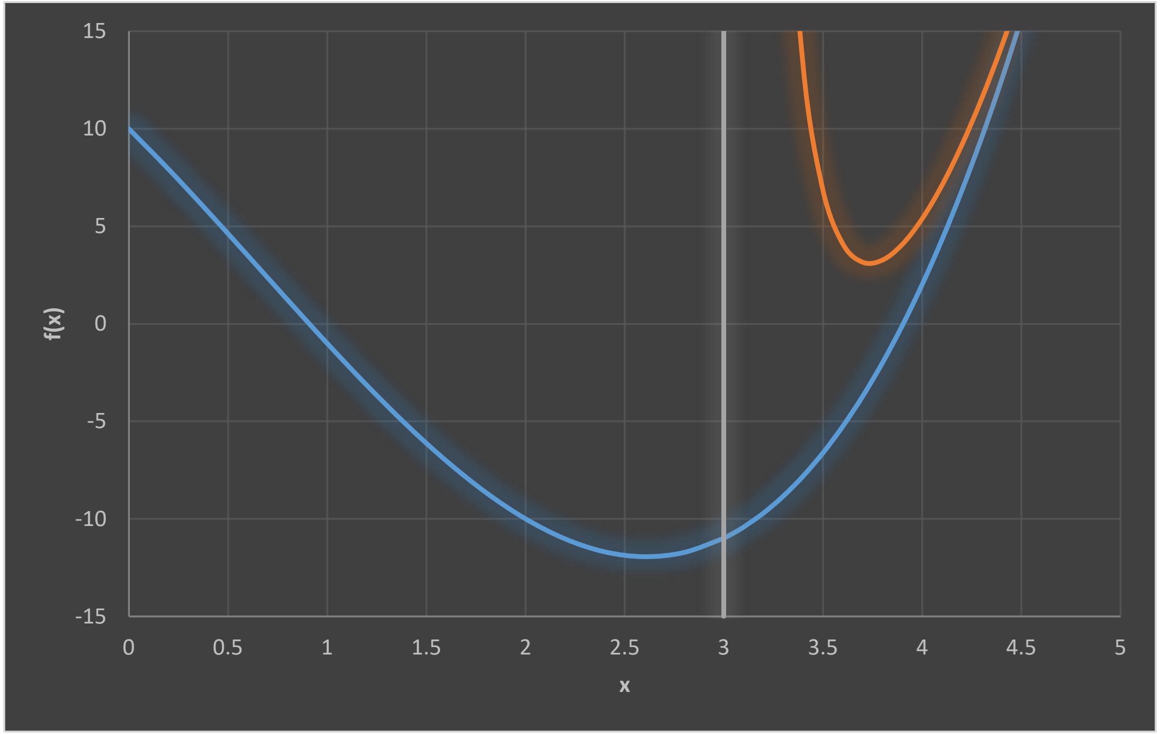


Minimize

$$f(x) = x^3 - 10x - 2x^2 + 10$$

Subject to $g(x) = x \geq 3$

Or, $g(x) = x - 3 \geq 0$

The problem can also be converted as

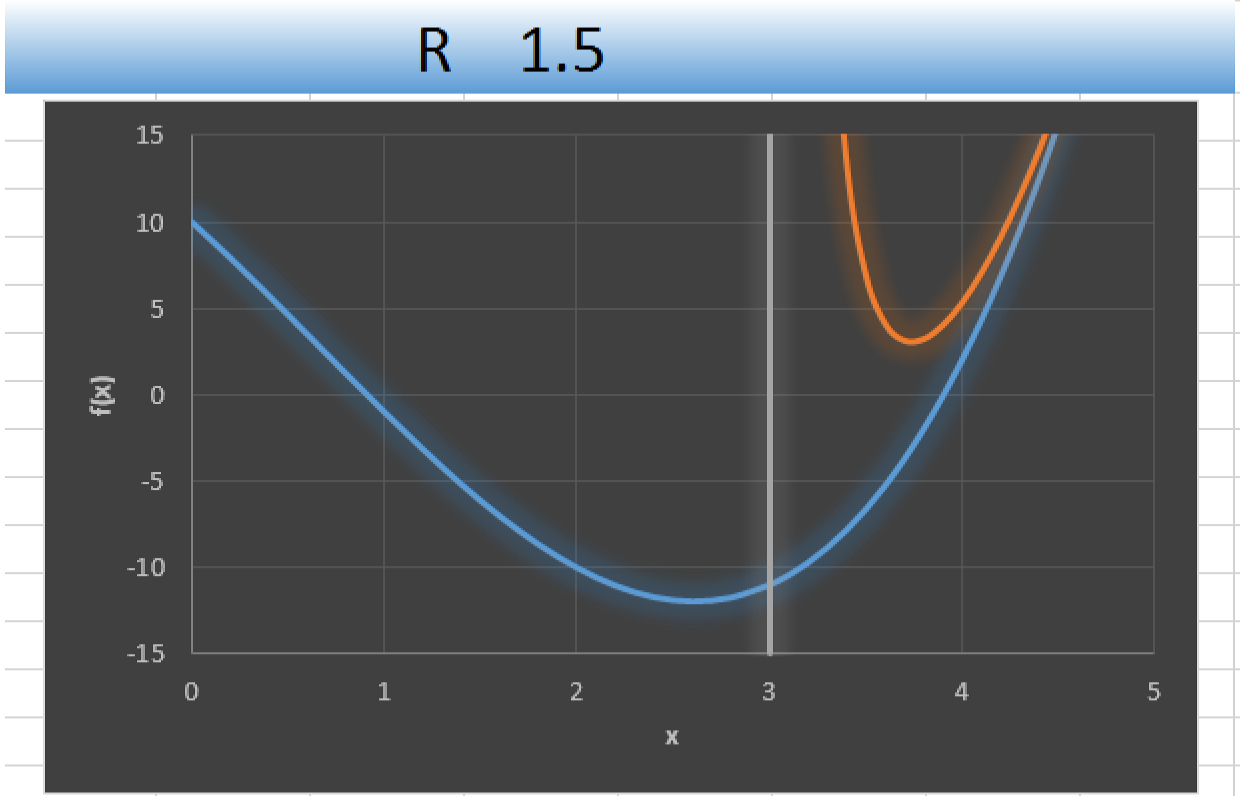
$$\text{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{g(x)}$$

This term is added in feasible side only

$$\text{F}(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{(x-3)}$$

Constrained Optimization

Minimize $F(x, R) = (x^3 - 10x - 2x^2 + 10) + R\frac{1}{(x-3)}$



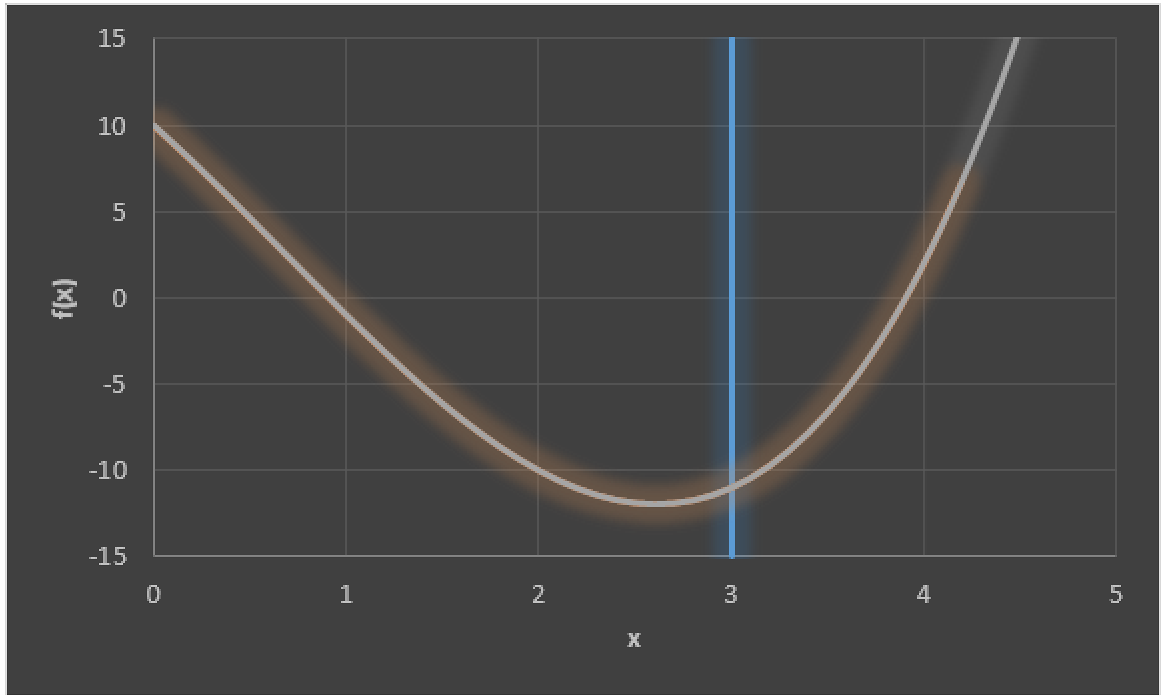
By changing R value, it is possible to avoid the infeasible solution

The minimization of the transformed function will provide the optimal solution which is in the feasible region only

Constrained Optimization

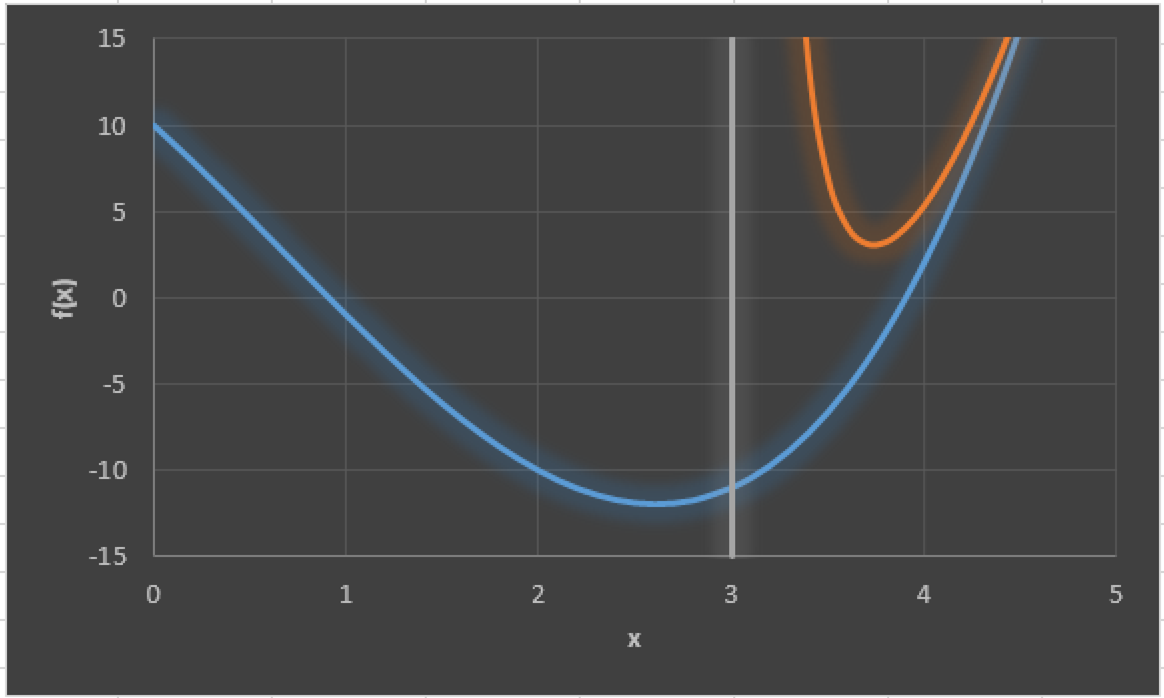
Exterior penalty method

R 0



Interior penalty method

R 1.5



Constrained Optimization

The transformation function can be written as

$$\mathrm{F}(X, R) = f(X) + \Psi\big(g(X), h(X)\big)$$

This term is called Penalty term

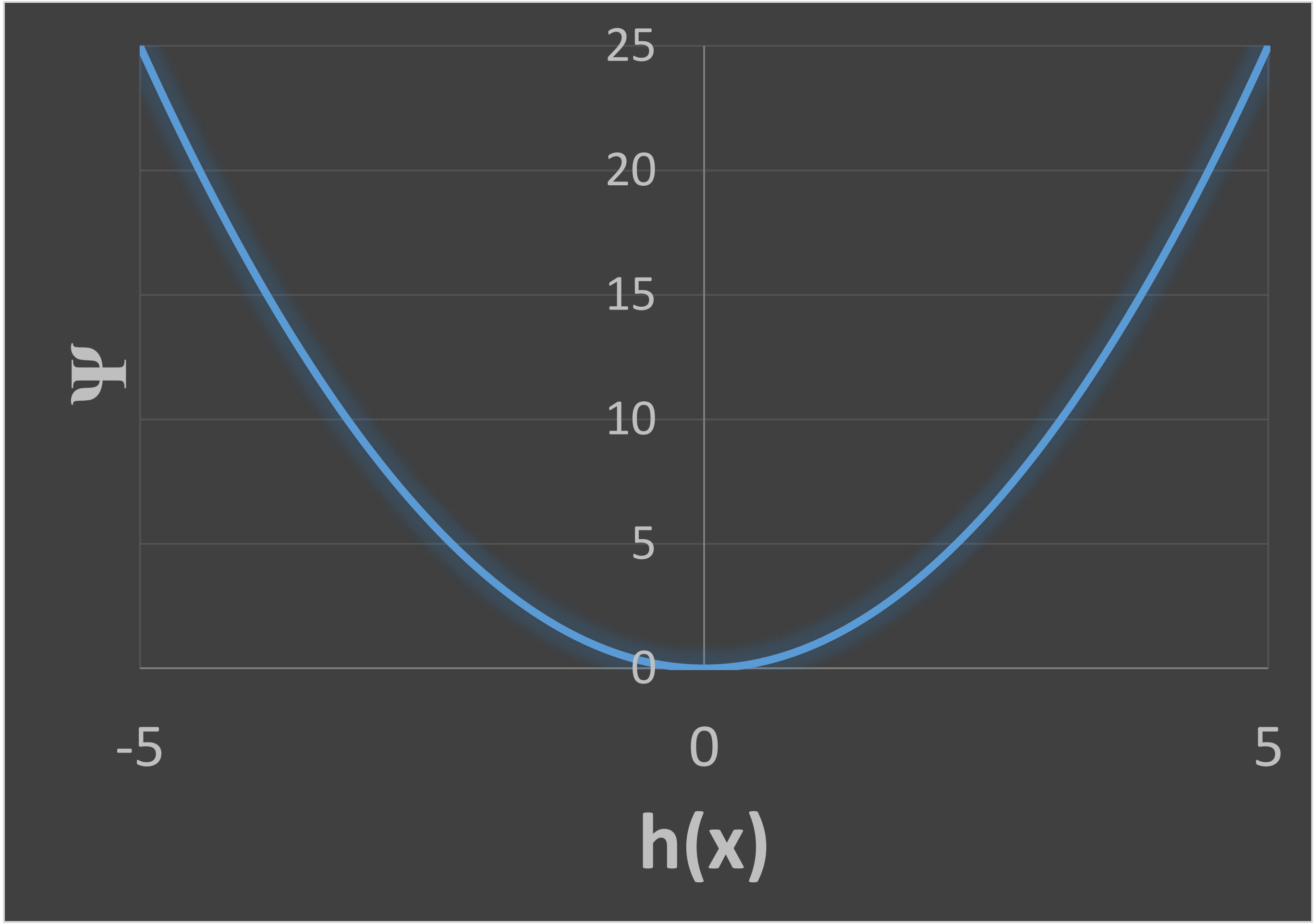
R Is called penalty parameters

Constrained Optimization

Penalty terms

Parabolic penalty

$$\Psi = R[h(x)]$$



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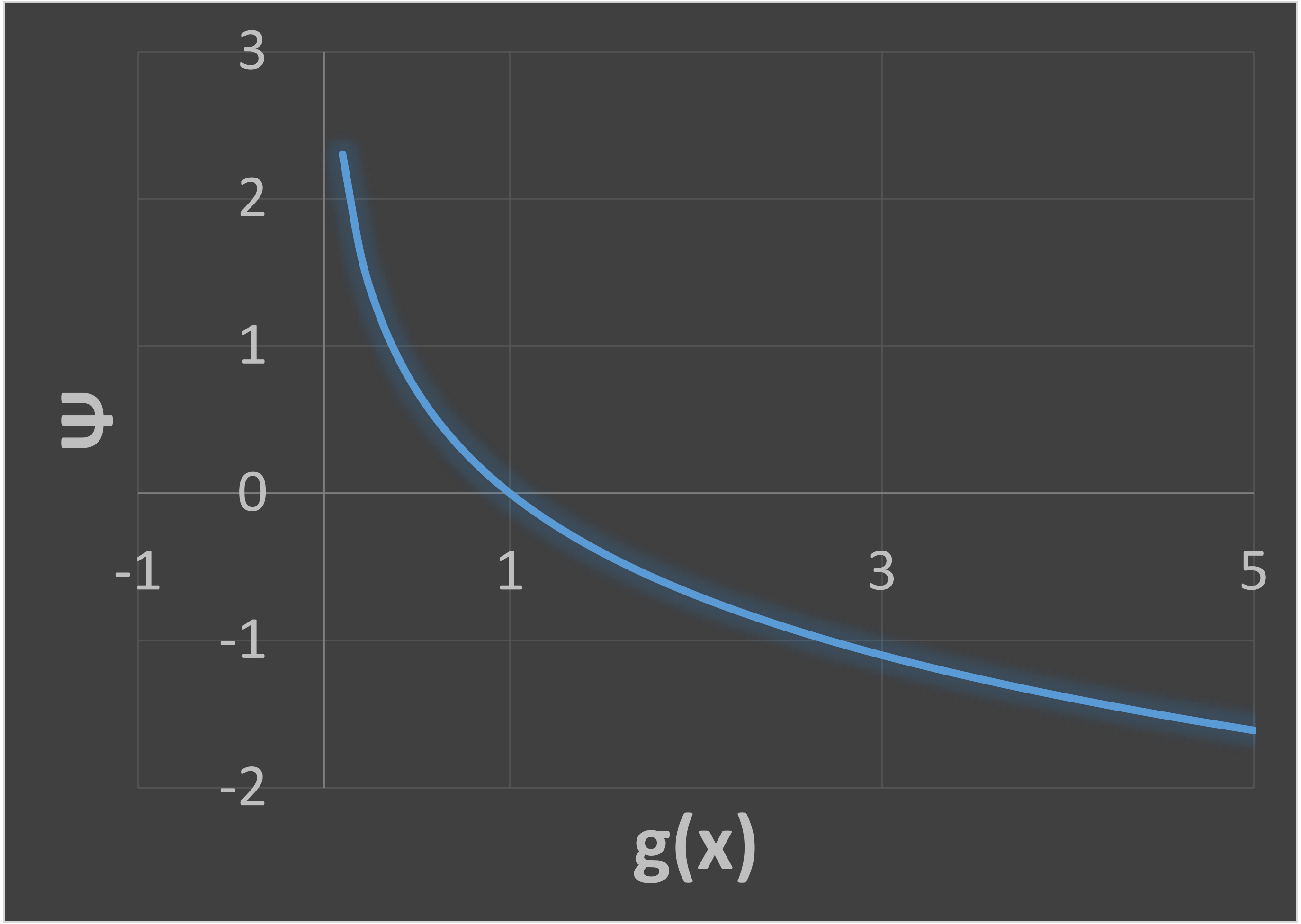
Penalty terms

Log penalty

$$\Psi = -R ln[g(x)]$$



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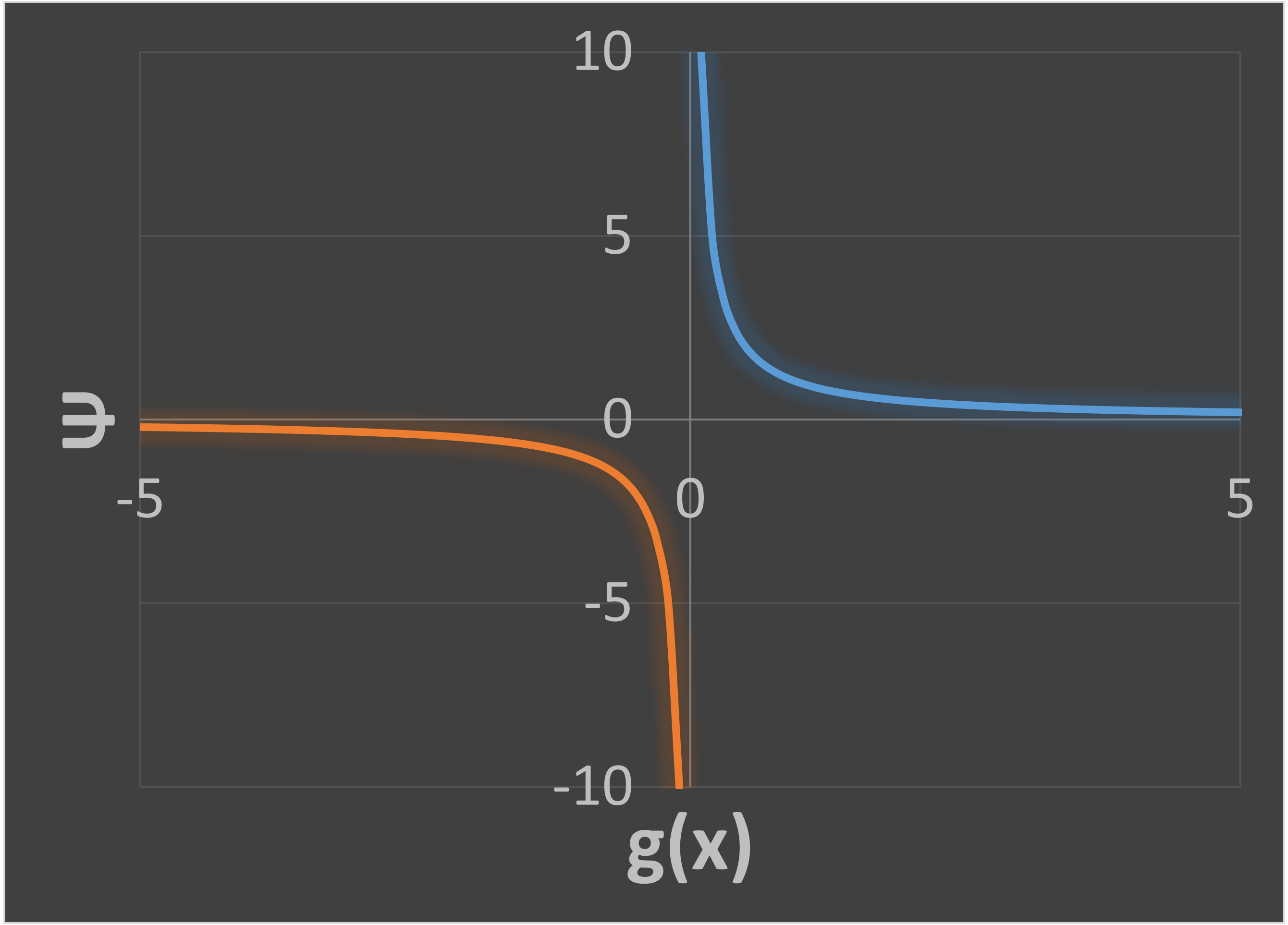
Penalty terms

Inverse penalty

$$\Psi = R\left[\frac{1}{g(x)}\right]$$



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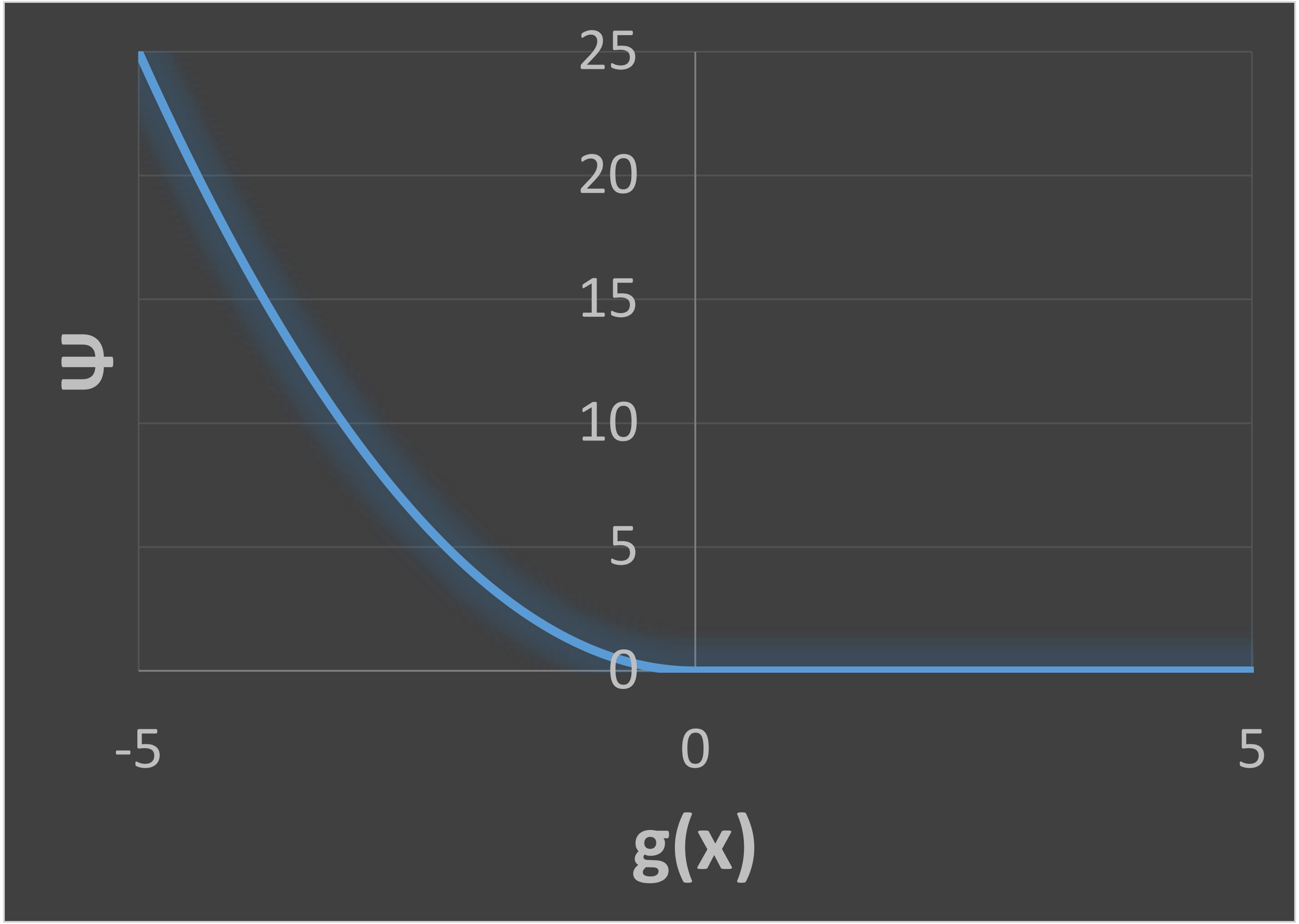
Penalty terms

Bracket operator

$$\Psi = R\langle g(x)\rangle$$



Constrained Optimization

Take an example

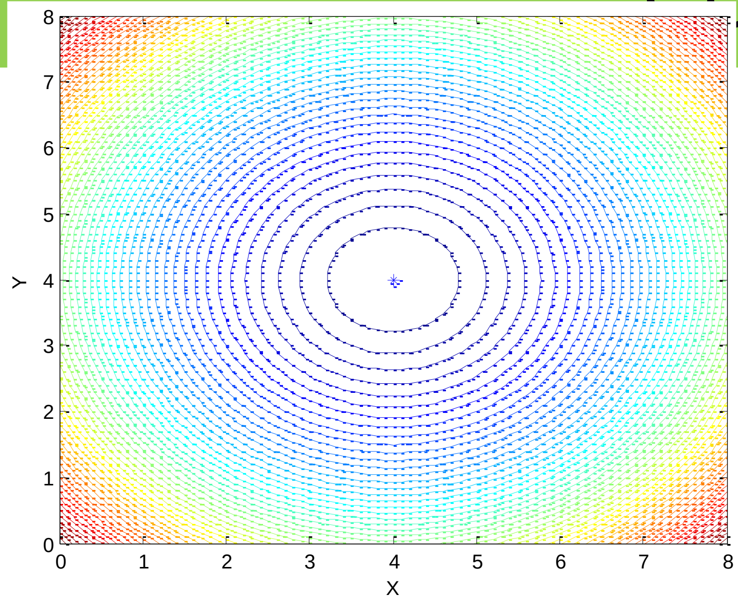
Minimize $f = (x_1 - 4)^2 + (x_2 - 4)^2$

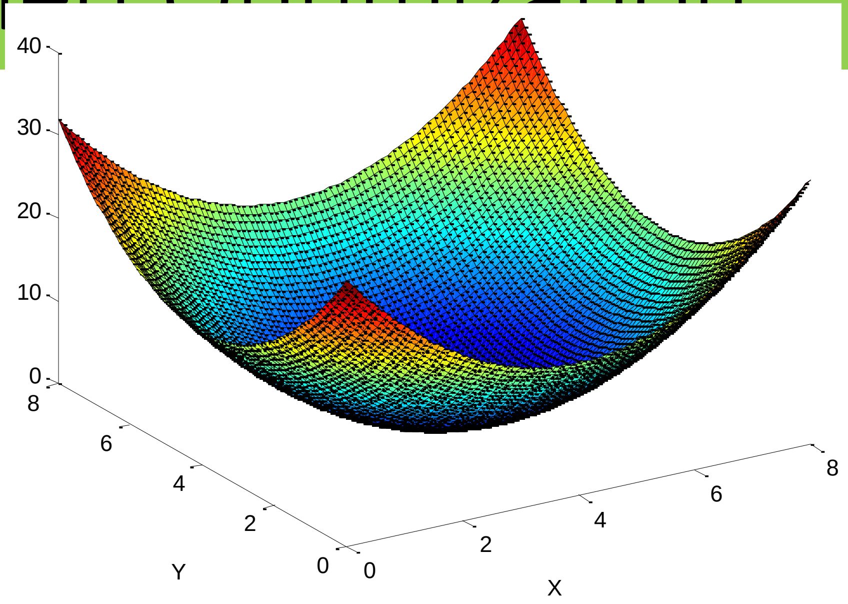
Subject to $g = x_1 + x_2 - 5 = 0$

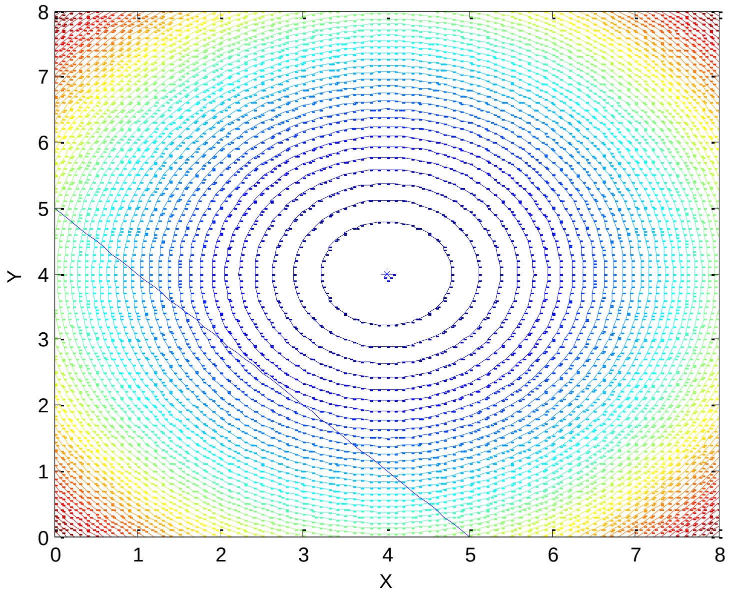
The transform function can be written as

Minimize $\mathrm{F} = (x_1 - 4)^2 + (x_2 - 4)^2 + R(x_1 + x_2 - 5\,)^2$

Constrained Optimization





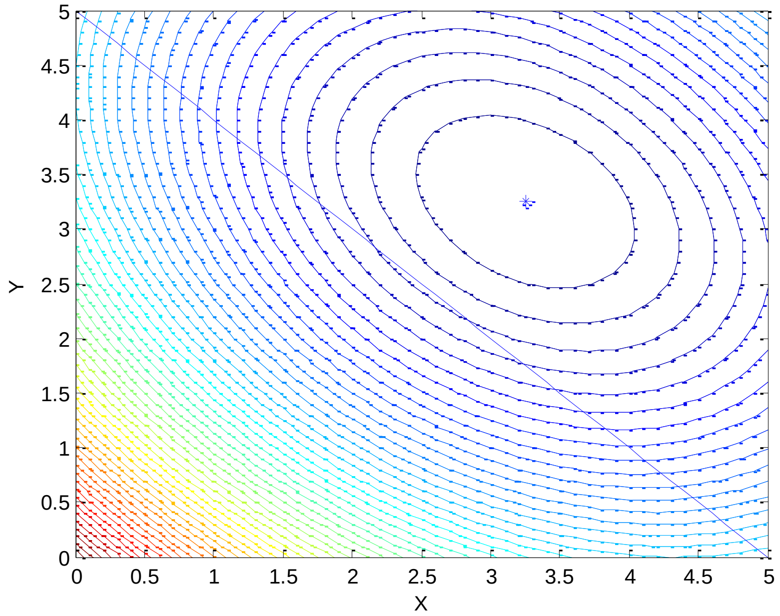


Minimize $f = (x_1 - 4)^2 + (x_2 - 4)^2$

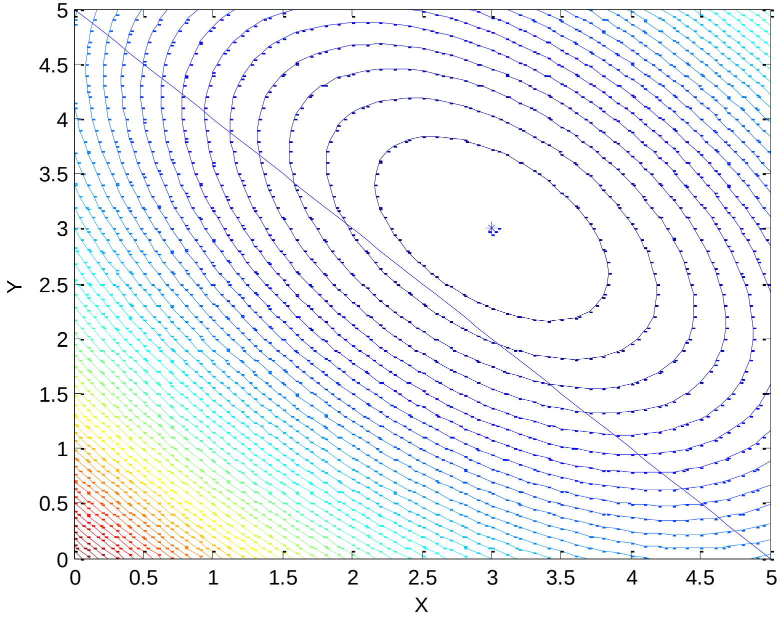
Subject to $g = x_1 + x_2 - 5 = 0$

Constrained Optimization

Minimize $F = (x_1 - 4)^2 + (x_2 - 4)^2 + R(x_1 + x_2 - 5)^2$



$R = 0.5$



$R = 1$

Optimal solution is

3.250	3.250

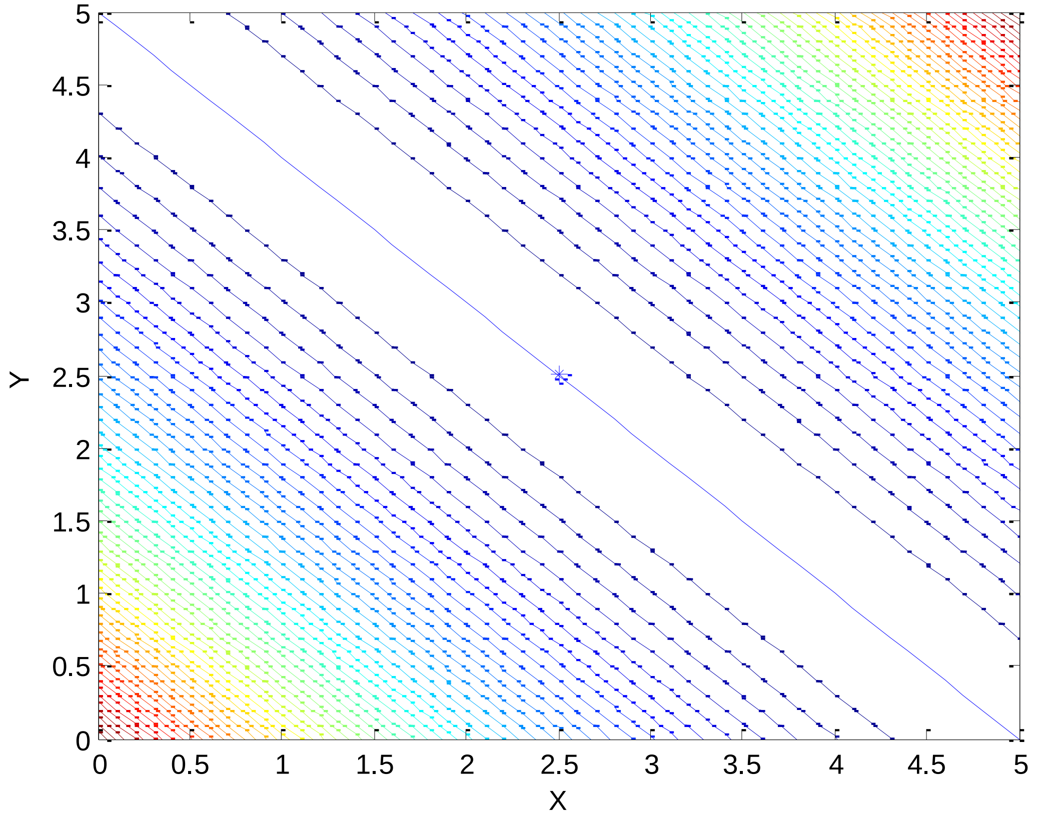
Optimal solution is

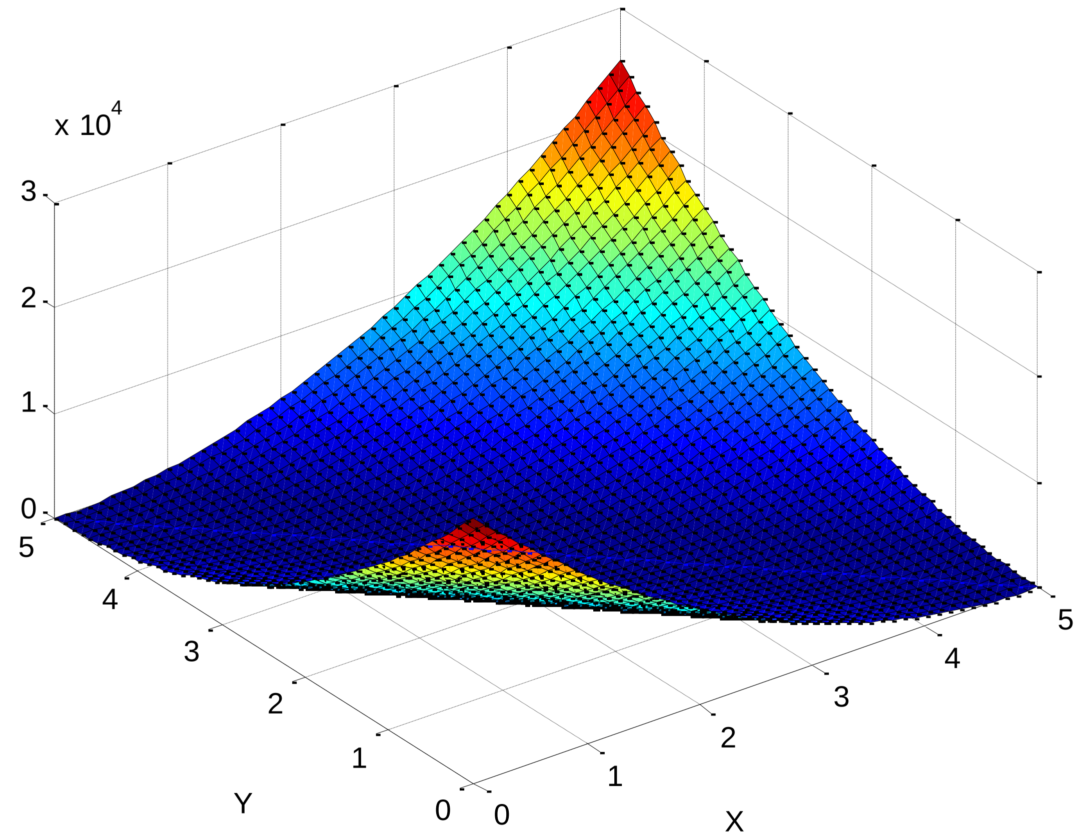
3.000	3.000

Constrained Optimization

R	x1	x2	f(x)	h(x)	F
0	4.000	4.000	0.000	3.000	0.000
0.5	3.250	3.250	1.125	1.500	2.250
1	3.000	3.000	2.000	1.000	3.000
5	2.636	2.636	3.719	0.273	4.091
10	2.571	2.571	4.082	0.143	4.286
20	2.537	2.537	4.283	0.073	4.390
30	2.525	2.525	4.354	0.049	4.426
50	2.515	2.515	4.411	0.030	4.455
100	2.507	2.507	4.455	0.015	4.478
200	2.504	2.504	4.478	0.007	4.489
500	2.501	2.501	4.491	0.003	4.496
1000	2.501	2.501	4.496	0.001	4.498
10000	2.500	2.500	4.500	0.000	4.500

Constrained Optimization





$R = 1000$

Optimal solution is | 2.501 | 2.501

Constrained Optimization

Thanks