## MA15010H: Multi-variable Calculus

(Assignment 3: Directional derivatives and differentiability) July - November, 2025

- 1. Let S be a nonempty open subset of  $\mathbb{R}^2$  and let  $f: S \to \mathbb{R}$  be such that the partial derivatives  $f_x$  and  $f_y$  exist at each point of S. If  $f_x: S \to \mathbb{R}$  and  $f_y: S \to \mathbb{R}$  are bounded, then show that f is continuous.
- 2. Find all  $u \in \mathbb{R}^2$  with ||u|| = 1 for which the directional derivative  $D_u f(0,0)$  exists (in  $\mathbb{R}$ ), if for all  $(x,y) \in \mathbb{R}^2$ ,

$$f(x,y) = \begin{cases} 1 & \text{if } y < x^2 < 2y, \\ 0 & \text{otherwise.} \end{cases}$$

- 3. State TRUE or FALSE with justification: If  $f: \mathbb{R}^2 \to \mathbb{R}$  is continuous such that all the directional derivatives of f at (0,0) exist (in  $\mathbb{R}$ ), then f must be differentiable at (0,0).
- 4. Determine all the points of  $\mathbb{R}^2$  where  $f:\mathbb{R}^2\to\mathbb{R}$  is differentiable, if for all  $(x,y)\in\mathbb{R}^2$ ,

$$f(x,y) = \begin{cases} x^{4/3} \sin\left(\frac{y}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

- 5. Let  $f: S \subseteq \mathbb{R}^m \to \mathbb{R}$  be differentiable at  $x_0 \in S^0$  and let  $f(x_0) = 0$ . If  $g: S \to \mathbb{R}$  is continuous at  $x_0$ , then show that  $fg: S \to \mathbb{R}$ , defined by (fg)(x) = f(x)g(x) for all  $x \in S$ , is differentiable at  $x_0$ .
- 6. Show that  $f: S \subseteq \mathbb{R}^2 \to \mathbb{R}$  is differentiable at  $(x_0, y_0) \in S^0$  iff there exist functions  $\varphi, \psi: S \to \mathbb{R}$  such that  $\varphi, \psi$  are continuous at  $(x_0, y_0)$  and  $f(x, y) f(x_0, y_0) = (x x_0)\varphi(x, y) + (y y_0)\psi(x, y)$  for all  $(x, y) \in S$ .
- 7. Let the temperature T(x,y) at any point  $(x,y) \in \mathbb{R}^2$  be given by  $T(x,y) = 2x^2 + xy + y^2$ . An insect is at the point (1,1).
  - (a) What is the best direction for the insect to move to feel cooler?
  - (b) In which direction should the insect move to feel no change in temperature?