MA201: Complex Analysis

Assignment 4

(Cauchy's theorem and Cauchy's integral formula)

July - November, 2024

1. Show that $\int\limits_{\gamma} \frac{e^{az}}{z^2+1}dz = 2\pi i \sin a$, where $\gamma(t) = 2e^{it},\ t \in [0, 2\pi]$.
2. Evaluate $\int\limits_0^{2\pi} e^{e^{i\theta}} d\theta$.
3. Let $f : \mathbb{C} \to \mathbb{C}$ be a function which is analytic on $\mathbb{C} \setminus \{0\}$ and bounded on $B(0, \frac{1}{2})$. Show that $\int\limits_{|z|=R} f(z)dz = 0$ for all $R > 0$.
4. Show that an entire function f satisfying $f(z+1) = f(z)$ and $f(z+i) = f(z)$ for all $z \in \mathbb{C}$ is a constant.
5. Let $g(z)$ be an analytic in $B(0, 2)$. Compute $\int\limits_{|z|=1} f(z)dz$ if
$$f(z) = \frac{a_k}{z^k} + \cdots + \frac{a_1}{z} + a_0 + g(z)$$
where a_i's are complex constants.
6. Let f be an entire function such that $|f(0)| \leq |f(z)|$ for all $z \in \mathbb{C}$. Then either $f(0) = 0$ or f is constant.
7. Whether primitive (anti-derivative) of $\frac{1}{z}$ exists on $\mathbb{C} \setminus \{0\}$? If NO, then specify the maximal domain in $\mathbb{C}$ where primitive exists.
8. Show that for $m \neq -1$, the z^m has primitive on $\mathbb{C} \setminus \{0\}$.