DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati
MA201: Mathematics-III

Quiz - I (Complex Analysis)   Maximum Marks: 10
Date: 24 August 2024   Time: 10:00 A.M. to 11:00 A.M.

Instructions

- *Please ensure that your **roll no, name, and tutorial group** are correctly indicated on the answer sheet and question paper.*

- *It is important to note that bio-breaks are **prohibited** during the quiz examination.*

- *Please promptly **submit the answer** sheet to the designated examination room invigilators after completing the quiz.*

- *If any student is found with a **phone, calculator, or similar device**, it will be reported to the academic section for necessary **disciplinary** action.*

N.B. Answer without proper justification will attract zero mark.

---

1. Prove that any non-constant harmonic function on a non-empty open set $D \subseteq \mathbb{C}$ is infinitely differentiable (partial derivatives of all orders exist and are continuous on D). $\boxed{3}$

2. For $z = x + iy \in \mathbb{C}$, classify all entire functions $f(z) = u(x,y) + iv(x,y)$ that satisfy $u_y(x,y) = v_x(x,y)$. $\boxed{2}$

3. Let $f : \mathbb{C} \to \mathbb{C}$ be given by

$$f(z) = \begin{cases} z^2 \sin \dfrac{1}{z}, & z \neq 0; \\ \\ 0, & z = 0. \end{cases}$$

   Discuss the continuity of the function f at $z = 0$. $\boxed{1}$

4. Find all possible value(s) of $z \in \mathbb{C}$ satisfying the equation $2i = \dfrac{1 + e^{2z}}{\cos(iz)}$. $\boxed{1}$

5. If g is an entire function satisfying $|g(z) - 2z| \leq 1$ on $|z| = 1$, show that $|g'(0)| \leq 3$. $\boxed{3}$

PAPER ENDS