DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA642: Real Analysis - I
Instructor: Rajesh Srivastava
Duration: 1.5 hours

Quiz II
Date: November 7, 2025
Maximum Marks: 10

Note: Answers lacking rigorous justification will not be awarded marks.

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1. (a) Does there exist a nested sequence of nonempty closed sets in $\mathbb{R}$ whose intersection is empty? **1**

   (b) Suppose $f_n \in C[0,1]$ is uniformly equicontinuous and $f_n \to f$ pointwise. Does it imply, $f_n \to f$ uniformly? **1**

2. Suppose f is a uniformly continuous function on a totally bounded metric space X. Show that $f(X)$ is totally bounded. **2**

3. For some fixed $f : \mathbb{R} \to \mathbb{R}$, define $f_n(x) = f(nx)$, for each $x \in \mathbb{R}$. If f_n forms an equicontinuous family, then show that f must be constant. **2**

4. Let $f_n \in C[0,1]$ be uniformly bounded. Define $F_n(x) = \int_0^x f_n(s)ds$. Show that F_n has a uniformly convergent subsequence. **2**

5. Show that $f_n : [0,1] \to \mathbb{R}$ defined by $f_n(x) = \dfrac{2nx}{1+n^2x^2}$ do not form an equicontinuous family at 0. **2**

END