## DEPARTMENT OF MATHEMATICS INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA642: Real Analysis - I

Instructor: Rajesh Srivastava

Duration: 1.5 hours

Quiz I

Date: September 1, 2025

Maximum Marks: 10

**Note:** Answers lacking rigorous justification will not be awarded marks.

- 1. (a) Is it necessarily true that every monotone and continuous function defined on  $\mathbb{R}$  is uniformly continuous?
  - (b) Consider the set  $\{f \in C[0,1] : ||f||_1 \le 1\}$ . Is this set bounded in the normed linear space  $(C[0,1], ||\cdot||_2)$ ?
- 2. Let  $f: \mathbb{R} \to \mathbb{R}$  be a monotone decreasing and bounded function. Define

$$h(x) = \inf\{f(y) : y < x\}.$$

Prove that if f is continuous at  $a \in \mathbb{R}$ , then

$$f(a) = h(a)$$
.

1

3. Let  $f(x) = e^{-x^2}$ . Show that for each  $n \in \mathbb{N}$ , there exists a constant M > 0, independent of x, such that

$$|f(x)| \le \frac{M}{(1+x^{2n})^2}.$$

Is it possible for M to be chosen independent of n as well?

4. Let  $f: \mathbb{R} \to \mathbb{R}$  be continuous, and suppose

$$\lim_{|x| \to \infty} f(x) = \infty.$$

Prove that f is bounded from below and attains its infimum.

5. Let  $0 \neq x \in \ell^{p_0}$  for some  $p_0 \geq 1$ . Show that

$$\lim_{p \to \infty} \sup ||x||_p \le ||x||_{\infty}.$$

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|2|

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