

## MA 101 (Mathematics-I)

### Multivariable Calculus part 2: Tutorial Problem Sheet 1

1. If  $f : D = [a, b] \times [c, d] \rightarrow \mathbb{R}$  is continuous, then  $f$  is uniformly continuous.
2. Let  $f$  be real valued continuous function on  $[a, b]$ . Show that the graph of  $f$  is a set of content zero.
3. Let  $D = \{(x, y) : a \leq x \leq b \text{ and } \varphi(x) \leq y \leq \psi(x)\}$ , where  $\varphi$  and  $\psi$  are continuous functions on  $[a, b]$ . If  $f$  is a bounded continuous functions on  $D$ , then

$$\iint_D f(x, y) dx dy = \int_a^b \left( \int_{\varphi(x)}^{\psi(x)} f(x, y) dy \right) dx.$$

4. Evaluate the following integral applying Fubini's Theorem

(a)  $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2} dy dx$

(b)  $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dy dx$

(c)  $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$

5. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function. Show that  $\int_{y=0}^x \int_{t=0}^y f(t) dt dy = \int_{t=0}^x (x-t)f(t) dt$ .
6. Let  $f$  be a continuous function on the bounded domain  $D$ . If  $\iint_R f(x, y) dx dy = 0$  for all rectangle  $R$  in  $D$ , then  $f = 0$  on  $D$ .
7. Let  $f : D = [a, b] \times [c, d] \rightarrow \mathbb{R}$  be a continuous function. If  $f_x, f_y, f_{xy}$  and  $f_{yx}$  are continuous then, by using Fubini's theorem, show that  $f_{xy} = f_{yx}$ .