MA 101 (Mathematics-I)

Multivariable Calculus part 2: Tutorial Problem Sheet 3

1. Find the line integral of the vector field $F(x, y, z) = y\vec{i} - x\vec{j} + \vec{k}$ along the path $c(t) = (\cos t, \sin t, \frac{t}{2\pi})$, $0 \le t \le 2\pi$ joining $(1, 0, 0)$ to $(1, 0, 1)$.

2. Evaluate $\int\limits_C T \cdot dR$, where C is the circle $x^2 + y^2 = 1$ and T is the unit tangent vector.

3. Show that the integral $\int\limits_C yzdx + (xz + 1)dy + xydz$ is independent of the path C joining $(1, 0, 0)$ and $(2, 1, 4)$.

4. Use Green's Theorem to compute $\int\limits_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the region $\{(x, y) : x, y \ge 0 \;\&\; x^2 + y^2 \le 1\}$.

5. If C is any simple closed and smooth curve in $\mathbb{R}^2$ which is not passing through the point $(1, 0)$, then evaluate the integral $\int\limits_C \frac{-ydx+(x-1)dy}{(x-1)^2+y^2}$.

6. Let $D = \{(x, y) : \; x^2 + y^2 < 1\}$. If $f : D \to R^2$ is a continuously differentiable function such that $\int\limits_\Gamma f \cdot dR = 0$ for every curve Γ in D, then f constant.

7. Use Stokes' Theorem to evaluate the line integral $\int\limits_C -y^3dx + x^3dy - z^3dz$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$ and the orientation of C corresponds to counterclockwise motion in the xy-plane.

8. Let $\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and let S be any surface that surrounds the origin. Prove that $\iint\limits_S \vec{F} \cdot n d\sigma = 4\pi$.

9. Let D be the domain inside the cylinder $x^2 + y^2 = 1$ cut off by the planes $z = 0$ and $z = x + 2$. If $\vec{F} = (x^2 + ye^z, y^2 + ze^x, z + xe^y)$, use the divergence theorem to evaluate $\iint\limits_{\partial D} F \cdot \hat{n}\, d\sigma$.