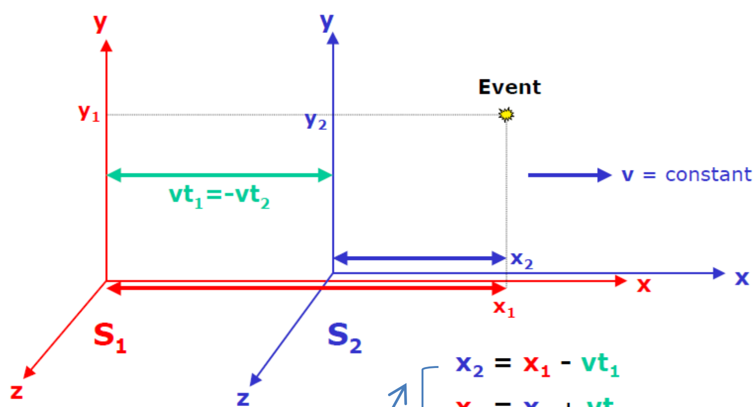
Special Theory of Relativity

PH101

Lec-2

Newtonian Relativity !

- The transformation laws are essential if we are to compare the mathematical statements of the laws of physics in different inertial reference frames.
  - ✓The transformation equations are the mathematical basis on which it can be shown that Newton's Laws are consistent with the principle of relativity.



$$\begin{cases} x_2 = x_1 - vt_1 \\ x_1 = x_2 + vt_2 \\ t_1 = t_2 \end{cases}$$

Inverse transform !!

These equations together are known as the Galilean Transformation !

Consider an inertial frame of reference S_1 *and a second* reference frame S_2 *moving with a velocity v relative to S .*

Let us suppose that the clocks in S_1 *and* S_2 *are set such that when the origins of the two reference* frames *coincide, all the clocks in both frames of reference read zero i.e.* $t_1 = t_2 = 0$. According to 'common sense', if the clocks in S_1 *and* S_2 *are synchronized at* $t_1 = t_2 = 0$, *then* they will always read the same, i.e. $t_1 = t_2$ *always.*

An event occurs with spatial coordinates (x_2, y_2, z_2) *at time* t_2 *in* S_2 *and at* (x_1, y_1, z_1) *at time* t_1 *in* S_1

Newtonian Relativity !

- Now suppose that in inertial frame S_1, a particle is acted on by no forces and hence is moving along the straight line path given by: $\mathbf{r}_1 = \mathbf{r}_0 + \mathbf{u}_1 t_1$

  - ✓Then in S_2, the particle will be following a path $\mathbf{r}_2 = \mathbf{r}_0 + (\mathbf{u}_1 - \mathbf{v})\, t_1$

    Particle moving in a straight line path at constant speed

  - ✓Incidentally, if we take the derivative of the above equation with respect to t_2, and use the fact that $t_1 = t_2$, we obtain $\mathbf{u}_2 = \mathbf{u}_1 - \mathbf{v}$

    Addition law for relative velocities

- We can now proceed further and study whether or not Newton's 2nd law of motion is ($\mathbf{F} = m\,\mathbf{a}$) indeed consistent with the principle of relativity.

There are in fact an infinite number of inertial frames of reference and it is of considerable importance to understand what happens to Newton's Second Law if we measure the force, mass and acceleration of a particle from different inertial frames of reference. In order to do this, we must make use of the Galilean transformation to relate the coordinates (x_1, y_1, z_1, t_1) of a particle in one inertial frame S_1 say to its coordinates (x_2, y_2, z_2, t_2) in some other inertial frame S_2 !

Newtonian Relativity !

- By means of the Galilean Transformation, we can obtain an important result of Newtonian mechanics which carries over in a much more general form to special relativity.

✓We shall illustrate the idea by means of an example involving two particles connected by a spring with spring constant k.

- Let the *X* coordinates of the two particles are x_1 and x_2 relative to *some* reference frame S -> The equation of motion of the particle at x_1 *is*

$$m_1 \frac{d^2 x_1}{dt^2} = -k(x_1 - x_2 - l)$$

✓ℓ is the natural length of the spring, and m_1 the mass of the particle.

✓Now let us look at the system from the point of view of another frame of reference S' moving with a velocity v_x relative to S, then

$$x_1 = x_1' + v_x t' \quad \text{and} \quad x_2 = x_2' + v_x t' \Rightarrow \frac{d^2 x_1}{dt^2} = \frac{d^2 x_1'}{dt'^2}$$

$$x_2 - x_1 = x_2' - x_1'$$

Therefore, $m_1' \frac{d^2 x_1'}{dt'^2} = -k(x_1' - x_2' - l)$, which is exactly same equation as obtained in **S** !

$\Rightarrow$ *Newton's Laws of motion are identical in all inertial frames of reference.*

Failure of Newtonian Relativity !

- Clash with electromagnetism !!

we won't go into here

✓Newton's laws reigned supreme in mechanics for more than 200 years

✓ However, difficulties arose in the mid-19th century with studies of electromagnetism

✓All electrical and magnetic effects could be summarised nicely in *Maxwell's Laws*

✓Does this mean that the principle of relativity is not valid for electricity or magnetism ?

Therefore, in a moving spaceship, it seemed that electromagnetic (including optical) phenomena would be different than they would be in a laboratory that was "at rest".

➢ Can you imagine, for example, if all of the magnets on the space shuttle were to get weaker in the first half of its orbit and stronger again in the second half, just because it was moving in different directions through space as it went around the Earth? Something is wrong!

✓In particular, the conflict become apparent where Maxwell's Laws predicted a *constant speed of light, independent of the* speed of the source.

Failure of Newtonian Relativity !

- Maxwell's equations predict the existence of e.m. waves propagating through free space with speed c = 3.00×10^8 m/s !

Question #1: With respect to what frame is c to be measured?

Question #2: Through what medium do e.m. waves propagate?

✓ 19th century answer: "Ether" hypothesis

Light must need a medium to propagate. Remember that nobody had come across the idea of a wave without a medium until then....

✓ The Victorian scientists named this medium the *ether …*

✓ It had to have some strange properties:

- Invisible
- It was massless
- It filled all of space
- High rigidity, so light could travel so quickly through it
- It had no drag on objects moving through it

Failure of Newtonian Relativity !

✓ Electromagnetic waves propagate through the ether
✓ c is to be measured with respect to the ether

An experiment is needed to test the presence of ether!

- **Michelson-Morley Experiment**

✓**Basis:** If one believes in the ether **(and that c is $3x10^8$ w.r. to it**), and in Maxwell's Equations, and in Galilean transformations for electromagnetic theory, then if one measures the speed of light in a frame moving w.r. to ether, one should be able to measure **v** of this moving frame.

✓ **Intension**: Measurement of earth's speed as it moves through ether !

Simply stated, they argued that if light is moving with a velocity **c** through the ether, and the Earth was at some stage in its orbit moving with a velocity **v** relative to the ether, then light should be observed to be travelling with a velocity **c' = c – v** relative to the Earth *!*

✓ **Outcome : Null; can not detect any motion of the earth through ether !!**

✓They found that the speed of light was always the same !

Implications: Question: Do Maxwell's equations obey Newtonian Relativity? → No!

What's next ?

- Couple of points need to sorted out

  - ❑ Are Maxwell's equations not the same in all inertial frames?

    ✓Maxwell's equations could be wrong (not that old !!)

    If so, Galilean transformations would be retained and
    – Maxwell's equations should be fixed to obey Galilean transformation !
    – A preferred inertial frame must exist which should produce different values for **c** as one moves through it !

  - ➢Maxwell's equations proved to be totally successful in application !!

  - ❑ Are Galilean transformations wrong?

    ✓Then, keep Maxwell's equations as they are; experiments OK !!

    ✓Absolute space and absolute time would have to be abandoned !

The Galilean transformation, and the Newtonian principle of relativity based on this transformation are wrong and that there exist a new relativity principle valid for both mechanics and electromagnetism that is not based on the Galilean transformation !

Einstein's Relativity !

- <u>Based on two postulates</u>

1) All the laws of physics are the same in every inertial frame of reference. This postulate implies that there is no experiment whether based on the laws of mechanics or the laws of electromagnetism from which it is possible to determine whether or not a frame of reference is in a state of uniform motion.

2) The speed of light is independent of the motion of its source.

The velocity of light in empty space is the same in all reference frames, and is independent of the motion of the emitting body.

Einstein was inspired to make these postulates through his study of the properties of Maxwell's equations and not by the negative results of the Michelson-Morley experiment, of which he was apparently only vaguely aware. It is this postulate that forces us to reconsider what we understand by space and time.

Einstein's Relativity !

Problems solved by STR:

- Keep Maxwell's equations
- Accept measurements of **c**, all producing 3.00×10^8 m/s
- No need for either
    - build a theory on experimental results, not the other way around!

New issues arise:

- Galilean transformations are abandoned
    - need new transformation equations
- Newton's laws don't obey Postulate 1 any longer
    - have to be changed
    - Absolute space and absolute time must be abandoned
    - Constancy of mass (and conservation of mass) must be abandoned
    - Laws are OK the way they are as long as $v << c$

Evaluation of Time

- <u>In Newtonian physics we previously assumed that t = t'</u>

  ✓ *with "synchronized" clocks, events in S and S' can be considered simultaneous !*

- ❑ If c is constant then space and time become relative !!

  ✓ We will be lead to some very interesting and unexpected conclusions regarding time intervals and distance measurements in inertial reference frames.

  - different observers can measure different lengths for the same object, or different times for the same event.

- ❑ Einstein realized that each system must have its own observers with their own clocks and meter sticks !

  ✓ Therefore, events considered simultaneous in S may not be in S' !

- ❑ Before proceeding further with the consequences of these two rather innocent looking postulates, we have to be more precise about how we go about measuring time in an inertial frame of reference !