Quantum Mechanics
Tutorial-1

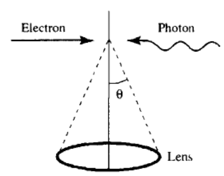
1) If a wavelength can be associated with every moving particle, then why are we not forcibly made aware of this property in our everyday experience? In answering, calculate the de Broglie wavelength of each of the following particles:

a) A person of mass 60 kg traveling at a speed of $1ms^{-1}$.

b) An N_2 molecule in air at temperature $300K$ (atomic mass of N = 2.33×10^{-26} kg).

c) An electron in an atom with energy E = 5 eV ($m_e = 9.11 \times 10^{-31}$ kg) .

2) Figure below describes schematically an experimental apparatus whose purpose is to measure the position of an electron. A beam of electrons of well defined momentum p_x moving in the positive x-direction scatters light shining along the negative x-axis. A certain electron will scatter a certain photon that will be detected through the microscope. According to the optics theory, the precision with which the electron can be localised is $\Delta x = \lambda / \sin\theta$, where λ is the wavelength of the light. Show that if we intend to minimize Δx by reducing λ, this will result in a loss of information about the x-component of the electron momentum.



3) Use the uncertainty relation to estimate the ground state energy of a harmonic oscillator. The energy is given by (Hint: the ground state is the minimum uncertainty state)

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

4) A localized wave packet in free space will spread due to its initial distribution of momenta. This wave phenomenon is known as dispersion. Consider a general wave packet in free space at time t = 0 whose characteristic width is given by a. Find the characteristic width in momentum p, energy E and the time scale t associated with the packet. The time t set the scale at which the packet will spread. Find this for a macroscopic object of mass $1g$ and width $1cm$. Comment on the result.

5)For gravity waves (deep in water), the relation between the frequency and wavelength is given by $\nu = \sqrt{g/(2\pi\lambda)}$ what are the group and phase velocities ?

6) For standard dice (ludo), what are the probabilities of finding each number. Calculate the average ($\bar{n}$) of all the measurements in terms of respective probabilities. The $\bar{n}$ provides no information about the spreading (uncertainty), the simplest possible measure of the uncertainty is given by $\Delta n = \sqrt{\overline{(n-\bar{n})^2}}$. Estimate Δn for the given case.

You may use the following inputs: Planck Constant $h = 6.63 \times 10^{-34}$ J.s, Boltzman constant $k_B = 1.38 \times 10^{-23}\ JK^{-1}$, $1eV = 1.6 \times 10^{-19}J$!