1. Finite sequence example to illustrate growth of terms where finite analysis is adequate: The size of economy of USA is \$23 trillion (Year: 2021), while that of India is \$3.17 trillion(Year 2021). US economy grows at a rate of 5%pa while India's economy grows at a rate of 9%. Will the Indian economy ever overtake US' economy? If so when?

   Hint:

Year	USA's size	India's size
2022	24.15	3.45
2031	37.26	7.48
2041	60.95	17.75
2051	99.36	42.03
2061	161.69	99.53
2071	263.58	235.68
2072	276.92	256.89
2073	290.72	280.03
2074	305.21	305.23
2075	320.39	332.72
2076	336.49	362.64

2. Pocket money: Would you rather have 1 crore rupees for a month or a paisa doubled every-day for a month?

n days in a month	Rupees: $\frac{1}{100}\sum_{k=0}^{n} 2^k$
28	26,84,354.55
29	53,68,709.11
30	1,07,37,418.23
31	2,14,74,836.47

   Hint:

3. A corrupt telecom minister has set up a kickback plan where he gets 1 Rupee for the first phone call, 1/2 of a Rupee for the second call, 1/3 of a rupee for the third and so on. Will he ever become a crorepati?

n	$\sum_{k=1}^{n} \frac{1}{k}$
1	1
10	2.928968254
100	5.1873775176
1000	7.4854708606
10000	9.787606036
100000	12.0901461299
1000000	14.3927267229
10000000	16.6953113659
100000000	18.9978964139
1000000000	21.3004815023
10000000000	22.064778

   Hint:

4. Zeno's Paradox: World famous runner Achilles who runs at a speed of 1Km/min is pitted against a tortoise which crawls at a speed of 100m/min. They start a race with the tortoise given a head start of 1Km. By the time (1min) Achilles runs 1Km, the tortoise has moved 100m. In the next 1/10 of a minute, Achilles covers the latter 100m, but the tortoise has moved 10m. In the next 1/100 of a minute, Achilles covers this 10m – but the tortoise covers another 1m. And so on for ever. If Achilles is always catching up with the marks left by tortoise at min., 1+1/10 of a minute, 1+1/10+1/100 of a minute and so on, how can Achilles ever overtake the tortoise?

5. Grandi's series: $1-1+1-1+1-1+\cdots$ What is the sum?

Zero? $(1-1)+(1-1)+(1-1)+\cdots = 0+0+0+\cdots = 0$

One? $1-(1-1)-(1-1)-(1-1)-\cdots = 1-0-0-0-\cdots = 1$,

Half? $S = 1-1+1-1+1-\cdots$ and $S = 1-(1-1+1-1+1-1+1-\cdots)$ implies $S = 1-S$ and $S = 1/2$

next Try $S_2 = 1-2+3-4+5-6+\cdots$. Add it to itself (with a shift) to get $S_2+S_2 = S_1$ so that S_2 equals 1/4.

more Take $S_3 = 1+2+3+4+\cdots$ and $S_3 - S_2 = 0+4+0+8+0+12+\cdots = 4S_3$ and hence $S_3 = -1/12$. Really?

6. Before you analyse the latter three questions, first prove that the multi–variable sum function $\Sigma : \mathbb{R}\times\mathbb{R}\times\cdots\times\mathbb{R} \to \mathbb{R}$ can be defined for finitely many variables. Using induction establish that this sum is both independent of ordering of the variables and the bracketing which dictates the partial sums.

7. The three questions raised, viz., Zeno's paradox and about the corrupt minister and the Grandi's series demand a definition for a sum of infinitely many reals. What would be your definition for the infinite sum $a_1 + a_2 + a_3 + \cdots$ where each a_n is real?

8. One can define a sequence of partial sums viz., $\sigma_1 = a_1$, $\sigma_2 = a_1 + a_2$, $\sigma_3 = a_1 + a_2 + a_3$, ... and see whether the sequence of numbers $\sigma_1, \sigma_2, \sigma_3, \ldots$ is getting *closer and closer* or *approaching* a fixed real number. This will be formalized via Cauchy's definition in the next lecture.

9. Discussion topics:

   (a) In how many ways can you add 2 numbers? 3 numbers? 4? 5? ...

   Answers: 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420, 24466267020, 91482563640, 343059613650, 1289904147324, 4861946401452,..